केन्द्रीय विद्यालय संगठन

KENDRIYA VIDYALAYA SANGATHAN

जयपुर संभाग / JAIPUR REGION

तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन

STUDY MATERIAL (MCQ's) FOR TERM -I

CLASS XII APPLIED MATHEMATICS (241)

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No.F. 230350(9)/2021-KVS(JPR)

Dated: 17/09/2021

MESSAGE

Preparation of this STUDY MATERIAL of Class XII Mathematics is the product of academic exercise undertaken by our learned teachers of Mathematics under the guidance and supervision of subject experts. This study material will provide the students a comprehensive, yet concise, learning support tool for consolidation of their studies. It consists of all the units in capsule form, mind maps, concepts with flow charts and MCQs from all the units to prepare students for Term – 1 examination.

The material has been prepared keeping in mind the latest CBSE curriculum and question paper design. This material provides the students a valuable window on precise information and covers all essential components that are required for effective revision of mathematics.

This material will surely prove to be a good tool for quick revision and will serve the purpose of enhancing student's confidence level to help them perform better. Planned study blended with hard work, good time management and sincerity will help the students reach the pinnacle of success.

17/9/2021

(B.L MORODIA)
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Applied Mathematics (Revision Material)

Class – XII **Term – 1 (2021 – 22)**

Numbers, Quantification and Numerical Applications

Topics in Syllabus

1. Modulo Arithmetic
2. Congruence Modulo
3. Allegation and Mixture
4. Numerical Problems (Boats and Streams, Pipes & Cisterns, Races & Games, Partnership)
5. Numerical Inequalities

➢ MODULO ARITHMETIC

For any four integers a, b, q and r, we have the relationship a = bq + r, $0 \le r \le |b|$.
Here a is dividend, b is divisor, q is quotient and r is the remainder.
Here we write a mod b = r [Example: 14 mod 5 = 4, -8 mod 5 = 2]
Congruence Modulo: When two numbers a and c both leave the same remainder when divided by b, we write a is congruent to c (mod b) or $a \equiv c \pmod b$.
Example: $37 \equiv 12 \pmod 5$ [Since both 37 and 12 leave remainder 2 when divided by 5]
37 – 12 = 25 is an integral of 5 that is $a \equiv c \pmod b \Rightarrow (a - c)$ is divisible by b.
Properties of Modulo Arithmetic

i. If a + b = c then a (mod n) + b (mod n) = c (mod n)
ii. If $a \equiv b \pmod n$ then a + k = (b + k) (mod n)
iii. If $a \equiv b \pmod n$ and If $c \equiv d \pmod n$ then (a + c) = (b + d) (mod n)
iv. If $a \equiv b \pmod n$ then If $-a \equiv -b \pmod n$.
v. If a . b = c then a (mod n) . b (mod n) = c (mod n)
vi. If $a \equiv b \pmod n$ then ka = kb(mod n)
vii. If $a \equiv b \pmod n$ and If $c \equiv d \pmod n$ then ac = bd (mod n)
viii. If $a \equiv b \pmod n$ then If $a^k \equiv b^k \pmod n$ ¥ k ϵ N

➢ ALLIGATION AND MIXTURE

Allegation means mixing two or more ingredients in some ratio.
Formula 1 : To find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture at desired price.

$$\boldsymbol{quantity\ of\ cheaper\ ingredient : quantity\ of\ dearer\ ingredient = d - m : m - c}$$

where c = cheaper price d = dearer price and m = mean price(desired)
Formula 2 : **(Repeated dilution)** – To calculate the pure quantity of a liquid left after 'n' number of repeated dilution.
Suppose a vessel contains 'x' units of liquid from which 'y' units are taken out and replaced by water, ***After 'n' number of repeated dilutions, the quantity of pure liquid =*** $\boldsymbol{x\left(1-\frac{y}{x}\right)^n}$ ***units.***

➢ NUMERICAL PROBLEMS

Boats and Streams: Let speed of boat in still water be x km/hr and speed of stream be y km/hr.
Then Downstream speed (boat rows in the direction of stream) = u = (x + y) km/hr
Upstream speed (boat rows in the opposite direction of stream) = v = (x – y) km/hr

i. $\boldsymbol{x = \frac{u+v}{2}}$ **and** $\boldsymbol{y = \frac{u-v}{2}}$

ii. $\frac{Time\ taken\ in\ upstream}{Time\ taken\ in\ downstream} = \frac{x+y}{x-y}$

iii. $Average\ speed = \frac{(x^2-y^2)}{x}$

iv. $Distance\ between\ two\ places, d = \frac{t(x^2-y^2)}{2x}$.

v. If a boat takes t hours more in upstream than downstream for covering the same distance d, then

$d = \frac{t(x^2-y^2)}{2y}$

- **PIPES AND CISTERNS**

Inlet is a pipe to fill the tank; outlet is a tank to empty the tank/reservoir.

Basic Concepts and Formulae

i. If a pipe can fill a tank in 'x' hours then part of tank filled in 1 hour = 1/x

ii. If a pipe can empty a tank in 'y' hours then part of tank emptied in 1 hour = 1/y

iii. A pipe can fill the tank in 'x' hrs and another pipe can empty it in 'y' hrs, if bothe pipers are opened simultaneously then

Case I: When y > x; The part of tank filled in 1 hr = $\frac{1}{x} - \frac{1}{y}$& time taken to fill the tank = $\frac{xy}{y-x}$

Case II: When x > y; The part of tank emptied in 1 hr = $\frac{1}{y} - \frac{1}{x}$& time taken to fill the tank = $\frac{xy}{x-y}$

iv. Two pipes can fill a tank in x and y hours respectively, if both are opened simultaneously then

The part of tank filled in 1 hr = $\frac{1}{x} + \frac{1}{y}$& time taken to fill the tank = $\frac{xy}{x+y}$.

v. Two pipes can fill a tank in x and y hours respectively and a third pipe can empty it in z hours then

The part of tank filled in 1 hr = $\frac{1}{x} + \frac{1}{y} - \frac{1}{z}$& time taken to fill the tank = $\frac{xyz}{yz+zx-xy}$.

RACES AND GAMES

Race is a competition to cover a fixed distance in the least time. A race starts with a starting point and is finished at Finish point/winning point/goal. Suppose there are two participants in a race, then

- *'A gives B a start of x meter'*means A starts the race from starting point and B starts from 'x' meter ahead from A. This way, A has to cover a distance 'x' meter more than B.
- *'A beats B by x meters'* means when A reach the finish point, B is x meters behind.
- *'A can give B 30 points'* means A scores 100 points and B scores 70 points.

PARTNERSHIP

When two or more persons join hands for a business and share its profit/loss, they are said to be partners.

Types of partners:

i. *Working partner*: A partner who invests money and manages the business also is known as working partner. A working partner gets some incentive/salary/profit share for his manual work.

ii. *Sleeping partner*: A partner who only invests money in the business but does not work for it.

Types of Partnership:

1. *Simple Partnership*: The partners invest equal money for different time period or unequal money for same time period.
   i. Equal money invested for different time period: The profit/loss is distributed in the ratio of time.
   ii. Unequal money invested for equal time period: The profit/loss is distributed in the ratio of money.
2. *Compound Partnership*: Partners invest unequal amount for different time periods. In this case the profit/loss is proportionate to the product of money invested and time.
   $\frac{Share\ of\ A\ in\ profit/loss}{Share\ in\ B\ in\ profit/loss} = \frac{xt_1}{yt_2}$; Where x & y is the money invested for time t_1 and t_2 by A and B respectively

NUMERICAL INEQUALITIES

The following mathematical signs represent the inequalities:

'< (Less than), > (Greater than), ≤ (Less than or equal to), ≥ (Greater than or equal to).

First two (<, >) are known as strict inequalities and the other two (≤, ≥) as slack inequalities.

Two real numbers or algebraic expressions connected with these signs are called an inequality.

Inequalities which do not involve variables are called Numerical Inequalities [Example: 2 < 3, 5/2 > 1]

Properties of Inequalities

1. Any number can be added/subtracted to/from both side of an inequality without changing its sign.
2. Any non-zero positive number can be multiplied/divided both side of an inequality without changing its sign. Sign is inverted if a negative number is multiplied/divided.
3. The rules of transferring a number from LHS to RHS or RHS to LHS in an inequality are same as in an equation.
4. The table given below shows the conclusions related to inequalities:

S.No.	Statement	Conclusion	S.No.	Statement	Conclusion
1	x = y + z, z > 0	x > y	5	A ≥ B ≥ C	A ≥ C
2	x = y + z, z < 0	x < y		A = B ≥ C	
3	A > B > C	A > C		A ≥ B = C	
	A > B ≥ C		6	A ≤ B ≤ C	
	A ≥ B > C			A = B ≤ C	

	A = B > C			A ≤ B = C	A ≤ C
	A > B = C		7	A < B < C	No Conclusion
4	A < B < C	A < C		A ≤ B < C	
	A < B ≤ C			A < B ≥C	
	A ≤ B < C			A > B < C	
	A = B < C			A ≥ B < C	
	A < B = C			A > B ≤ C	

ALGEBRA

Matrix : In mathematics, a matrix (plural matrices) is a rectangular array or table of numbers, symbols, or expressions, arranged in rows and columns, which is used to represent a mathematical object or a property of such an object. For example,

$\begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & 7 \end{bmatrix}$ is a matrix with two rows and three columns; one say often a "two by three matrix", a "2×3-matrix", or a matrix of dimension 2×3.

Types of matrices

Row matrix ; A matrix with one row, sometimes used to represent a vector $\begin{bmatrix} 1 & 2 \end{bmatrix}$

Column matrix ; A matrix with one column, sometimes used to represent a vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Square matrix : A matrix with the same number of rows and columns, sometimes used to represent a linear transformation from a vector space to itself, such as reflection , rotation.

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

Addition :

The *sum* A+B of two *m*-by-*n* matrices A and B is calculated entrywise:

$(A + B)_{i,j} = A_{i,j} + B_{i,j}$, where $1 \le i \le m$ and $1 \le j \le n$.

$$\begin{bmatrix} 1 & 3 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 5 \\ 7 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1+0 & 3+0 & 1+5 \\ 1+7 & 0+5 & 0+0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 6 \\ 8 & 5 & 0 \end{bmatrix}$$

Scalar multiplication : The product cA of a number c (also called a scalar) and a matrix A is computed by multiplying every entry of A by c:

$(cA)_{i,j} = c \cdot A_{i,j}$. This operation is called scalar multiplication

$$2 \cdot \begin{bmatrix} 1 & 8 & -3 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 2 \cdot 1 & 2 \cdot 8 & 2 \cdot -3 \\ 2 \cdot 4 & 2 \cdot -2 & 2 \cdot 5 \end{bmatrix} = \begin{bmatrix} 2 & 16 & -6 \\ 8 & -4 & 10 \end{bmatrix}$$

Transposition : The *transpose* of an *m*-by-*n* matrix A is the *n*-by-*m* matrix A^T formed by turning rows into columns and vice versa: $(A^T)_{i,j} = A_{j,i}$.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -6 & 7 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 0 \\ 2 & -6 \\ 3 & 7 \end{bmatrix}$$

Matrix multiplication : *Multiplication* of two matrices is defined if and only if the number of columns of the left matrix is the same as the number of rows of the right matrix. If A is an *m*-by-*n* matrix and B is an *n*-by-*p* matrix, then their *matrix product* AB is the *m*-by-*p* matrix whose entries are given by dot product of the corresponding row of A and the corresponding column of B

where $1 \le i \le m$ and $1 \le j \le p$

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1000 \\ 1 & 100 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 3 & 2340 \\ 0 & 1000 \end{bmatrix}.$$

Matrix multiplication satisfies the rules (AB)C = A(BC) (associativity), and (A + B)C = AC + BC as well as C(A + B) = CA + CB (left and right distributivity), whenever the size of the matrices is such that the various products are defined.The product AB may be defined without BA being defined, namely if A and B are *m*-by-*n* and *n*-by-*k* matrices, respectively, and $m \neq k$. Even if both products are defined, they generally need not be equal, that is: $AB \neq BA$. In other words, matrix multiplication is not commutative.

Row operations

There are three types of row operations:

1. row addition, that is adding a row to another.
2. row multiplication, that is multiplying all entries of a row by a non-zero constant;
3. row switching, that is interchanging two rows of a matrix;

Square matrix :

A square matrix is a matrix with the same number of rows and columns. An *n*-by-*n* matrix is known as a square matrix of order *n.* Any two square matrices of the same order can be added and multiplied. The entries a_{ii} form the main diagonal of a square matrix.

Identity matrix ; The *identity matrix* I_n of size *n* is the *n*-by-*n* matrix in which all the elements on the diagonal are equal to 1 and all other elements are equal to 0, for example,

$$\mathbf{I}_1 = [1],\ \mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \ldots,\ \mathbf{I}_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Symmetric or skew-symmetric matrix:

A square matrix A that is equal to its transpose, that is, $A = A^{\mathrm{T}}$, is a symmetric matrix . If instead, A is equal to the negative of its transpose, that is, $A = -A^{\mathrm{T}}$, then A is a skew symmetric matrix.

Invertible matrix and its inverse ;

A square matrix A is called invertible or *non-singular* if there exists a matrix B such that $AB = BA = I_n$, where I_n is the $n \times n$ identity matrix with 1s on the diagonal and 0s elsewhere. If B exists, it is unique and is called the inverse matrix of A, denoted A^{-1}.

DETERMINANT :

The *determinant* of a square matrix A (denoted det(A) or |A|) is a number encoding certain properties of the matrix. A matrix is invertible if and only if its determinant is nonzero.

The determinant of 2-by-2 matrices is given by

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc.$$

1. The determinant has several key properties that can be proved by direct evaluation of the definition for 2x2 -matrices, and that continue to hold for determinants of larger matrices. They are as follows:first, the determinant of the identity matrix is 1. Second, the determinant is zero if two rows are the same:
2. if the two columns are the same
$$\begin{vmatrix} a & b+b' \\ c & d+d' \end{vmatrix} = a(d+d') - (b+b')c = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a & b' \\ c & d' \end{vmatrix}.$$
3. if any column is multiplied by some number (i.e., all entries in that column are multiplied by that number), the determinant is also multiplied by that number
$$\begin{vmatrix} r \cdot a & b \\ r \cdot c & d \end{vmatrix} = rad - brc = r(ad - bc) = r \cdot \begin{vmatrix} a & b \\ c & d \end{vmatrix}.$$

Transpose : The determinant of the transpose of A equals the determinant of *A*

$$\det\left(A^{\mathsf{T}}\right) = \det(A)$$

Multiplication : For square matrices A and B of equal size, the determinant of a matrix productequals the product of their determinants:$|AB| = |A||B|$

Adjoint matrix : The adjoint matrix adjA is the transpose of the matrix of the cofactors, that is,

$$(\operatorname{adj}(A))_{ij} = (-1)^{i+j} M_{ji}.$$

For every square matrix A

$$(\det A)I = A \operatorname{adj} A = (\operatorname{adj} A)\, A.$$

Thus the adjoint matrix can be used for expressing the inverse of a non-singular matrix. $A^{-1} = \frac{1}{|A|}$adjA

System of linear equations:

A system of linear equations (or linear system) is a collection of one or more linear equations involving the same set of variables. For example

3x + 2y – z = 1

2x -2y + 4z = -2

-x + y – z = 0 is a system of three equations in the three variables *x*, *y*, *z*.

Matrix equation: The matrix equation is of the form Ax = b

where *A* is an *m*×*n* matrix, X is a column matrix with *n* entries, and b is a column matrix with *m* entries.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Solution set : A solution of a linear system is an assignment of values to the variables x_1, x_2, ..., x_n such that each of the equations is satisfied. The set of all possible solutions is called the solution set.

A linear system may have any one of three possible ways:

1. The system has *infinitely many solutions*.
2. The system has a single *unique solution*.
3. The system has *no solution*.

Consistency: A linear system is inconsistent if it has no solution, and otherwise it is said to be consistent

Solution of system of linear equations:

Crammer's rule: Cramer's rule is an explicit formula for the solution of a system of linear equations, with each variable given by a quotient of two determinants. For example, the solution to the system

$$\begin{aligned} x + 3y - 2z &= 5 \\ 3x + 5y + 6z &= 7 \\ 2x + 4y + 3z &= 8 \end{aligned}$$

is given by

$$x = \frac{\begin{vmatrix} 5 & 3 & -2 \\ 7 & 5 & 6 \\ 8 & 4 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & -2 \\ 3 & 5 & 6 \\ 2 & 4 & 3 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} 1 & 5 & -2 \\ 3 & 7 & 6 \\ 2 & 8 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & -2 \\ 3 & 5 & 6 \\ 2 & 4 & 3 \end{vmatrix}}, \quad z = \frac{\begin{vmatrix} 1 & 3 & 5 \\ 3 & 5 & 7 \\ 2 & 4 & 8 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & -2 \\ 3 & 5 & 6 \\ 2 & 4 & 3 \end{vmatrix}}.$$

For each variable, the denominator is the determinant of the matrix of coefficients, while the numerator is the determinant of a matrix in which one column has been replaced by the vector of constant terms.

Matrix method : If the equation system is expressed in the matrix form Ax = b , the entire solution set can also be expressed in matrix form. If the matrix *A* is square matrix and(all *m* rows are independent), then the system has a unique solution given by $x = A^{-1}b$

Where A^{-1} is inverse of coefficient matrix A and $\mathbf{A^{-1} = \frac{1}{|A|}adjA}$

ROW REDUCTION : In row reduction method, the linear system is represented as an augmented matrix:

For example

$$x + 3y - 2z = 5$$

$$3x + 5y + 6z = 7$$

$$2x + 4y + 3z = 8$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 3 & 5 & 6 & 7 \\ 2 & 4 & 3 & 8 \end{array}\right].$$ This matrix is then modified using elementary row operations until it reaches reduced **row echelon form**.

There are three types of elementary row operations:

1: Swap the positions of two rows.
2: Multiply a row by a nonzero scalar
3: Add to one row a scalar multiple of another.

Because these operations are reversible, the augmented matrix produced always represents a linear system that is equivalent to the original.

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 3 & 5 & 6 & 7 \\ 2 & 4 & 3 & 8 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 0 & -4 & 12 & -8 \\ 2 & 4 & 3 & 8 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 0 & -4 & 12 & -8 \\ 0 & -2 & 7 & -2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 0 & 1 & -3 & 2 \\ 0 & -2 & 7 & -2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 0 & 9 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -15 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 2 \end{array}\right].$$

The last matrix is in reduced row echelon form, and represents the system $x = -15$, $y = 8$, $z = 2$.

Leontief input output model :

The model depicts inter-industry relationships within an economy, showing how output from one industrial sector may become an input to another industrial sector. In the inter-industry matrix, column entries typically represent inputs to an industrial sector, while row entries represent outputs from a given sector. This format, therefore, shows how dependent each sector is on every other sector, both as a customer of outputs from other sectors and as a supplier of inputs. Each column of the input–output matrix shows the monetary value of inputs to each sector and each row represents the value of each sector's outputs.

Say that we have an economy with n sectors. Each sector produces x_i units of a single homogeneous good. Assume that the jth sector, in order to produce 1 unit, must use a_{ij} units from sector i. Furthermore, assume that each sector sells some of its output to other sectors (intermediate output) and some of its output to consumers (final output, or final demand). Call final demand in the ith sector d_i. Then we might write

$$x_i = a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n + d_i,$$

or total output equals intermediate output plus final output. If we let A be the matrix of coefficients a_{ij}, x be the vector of total output, and d be the vector of final demand, then our

expression for the economy becomes $x = Ax + d$

which after re-writing becomes $(I - A)x = d$. If the matrix I - A is invertible then this is a linear system of equations with a unique solution, and so given some final demand vector the required output can be found.

Example: Consider an economy with two goods, A and B. The matrix of coefficients and the final demand is given by

$$A = \begin{bmatrix} 0.5 & 0.2 \\ 0.4 & 0.1 \end{bmatrix} \text{ and } \mathbf{d} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}.$$

his corresponds to finding the amount of output each sector should produce given that we want 7 units of good A and 4 units of good B. Then solving the system of linear equations derived above gives us

$$\mathbf{x} = (I - A)^{-1}\mathbf{d} = \begin{bmatrix} 19.19 \\ 12.97 \end{bmatrix}.$$

CALCULAS

Some important formulae:

1. $\frac{d}{dx}(x^n) = nx^{n-1}$
2. $\frac{d}{dx}(sinx) = cosx$
3. $\frac{d}{dx}(cosx) = -sinx$
4. $\frac{d}{dx}(tanx) = sec^2x$
5. $\frac{d}{dx}(cotx) = -cosec^2x$
6. $\frac{d}{dx}(secx) = secx.tanx$
7. $\frac{d}{dx}(cosecx) = -cosecx.cotx$
8. $\frac{d}{dx}(a^x) = a^x loga$
9. $\frac{d}{dx}(logx) = 1/x$
10. $\frac{d}{dx}(sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$
11. $\frac{d}{dx}(cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$
12. $\frac{d}{dx}(tan^{-1}x) = \frac{1}{1+x^2}$
13. $\frac{d}{dx}(cot^{-1}x) = -\frac{1}{1+x^2}$
14. $\frac{d}{dx}(sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$
15. $\frac{d}{dx}(cosec^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$
16. $\frac{d}{dx}(e^x) = e^x$

Note:

1. Product Rule $(f.g)' = f.g' + g.f'$
2. Quotient Rule: $(\frac{f}{g})' = \frac{g.f' - f.g'}{g^2}$
3. Chain Rule:

If $y = f(u), u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$

4. $(f \pm g)' = f' \pm g'$

Application of derivatives

Increasing / decreasing of functions:

(i) If $f'(x) > 0$ for all $x \in (a,b)$, then f is increasing on (a,b)

(ii) If $f'(x) < 0$ for all $x \in (a,b)$, then f is decreasing on (a,b)

(iii) If $f'(x) = 0$ for all $x \in (a,b)$, then f is constant on (a,b).

Note:1. A function f is said to be ***monotonic*** on an interval, if it is either increasing or decreasing on that interval.

2. A function $y = f(x)$ is said to have a ***critical point*** at $x = c$, if any one of the following conditions is satisfied:

(a) $f'(c) = 0$ (b) $f'(c)$ is undefined, but $f(x)$ is continuous at $x = c$.

3. Let $y = f(x)$ be a given function. The points where $f'(x) = 0$ are called ***stationary/critical points*** of the function. So, we can find the stationary points of a function $y = f(x)$ by solving the equation $f'(x) = 0$ for x.

TANGENTS AND NORMALS

1)For the curve $y = f(x)$, $\frac{dy}{dx}$, represents the slope of the tangent to the curve. i.e $m = \frac{dy}{dx}$ *at the given point*

2) Slope of the tangent to the curve at (x_1, y_1) *is* $\frac{dy}{dx}](x_1, y_1)$.

3) Equation to the tangent at $(x_1, y_1)$to a curve$y = f(x)$ with slope m is $y - y_1 = m(x - x_1)$

4) If m is the slope of the tangent to the curve $y = f(x)$ at(x_1, y_1)then slope of the normal at (x_1, y_1)**is -1/m.**

5)Equation of normal at $(x_1, y_1)$is $y - y_1 = \frac{-1}{m}(x - x_1)$

6) Two lines are parallel if their slopes are equal i.e. m_1=m_2

7) Two lines are perpendicular if product of their slopes=-1 i.e. $m_1.m_2$=-1

8)The point where the equation of tangent or normal is asked is lie on the curve.

Maxima & Minima

1)There are two types of extreme positions: ***local (relative)*** and ***global (absolute).***

2)A function $f(x)$ defined on an interval $[a,b]$ is said to have a ***local (or relative) maxima*** at a point $x = c$, if $f(c) \geq f(c+h)$ for all sufficiently small negative as well as positive values of h. The function is said to have a ***local (or relative) minima*** at $x = c$, if $f(c) \leq f(c+h)$.

3)The point $x = c$, where $f'(c) = 0$ or $f'(c)$ does not exist, is called a ***critical point*** of the function $f(x)$.

4)A maximum or a minimum value of a function is also termed as ***extremum*** or ***extreme value*** of the function.

5) Let f be function defined in the closed interval I. If there exist a point 'a' in the interval I such that f(a) ≥ f(x) for every x ϵ I, then the function is said to attain absolute maximum at x = a, and f(a) is absolute maximum value.

6) Let f be function defined in the closed interval I. If there exist a point 'a' in the interval I such that f(a) ≤f(x) for every x ϵ I, then the function is said to attain absolute minimum at x = a, and f(a) is absolute minimum value.

7) To find the absolute maximum or minimum in [a, b] we have to find out the value at the end points of interval [a, b] i.e. f(a) and f(b) along with local maxima and minima.**In some cases Local maxima may be less than the local minima.**

Tests for maxima or minima:

1) First Derivative Test : If a function $f(x)$ has either local maxima or minima at a point $x=c$, then either $f'(c)=0$ or $f'(c)$ does not exist, i.e, $x=c$ is a critical point of the function. Of course, there may be functions for which $f(c)$ is not a local extremum, even when $x=c$ is a critical point of the function.

2) If f'(x) does not change its sign in the neighbourhood of 'x_1', then 'x_1' is neither point of local maxima nor local minima, then x_1 is called the point of inflexion.

- ***Second Derivative Test:*** Let $f'(c)=0$ for a given function $f(x)$ defined on (a,b). Then

  (i) $f''(c)<0 \Rightarrow f(c)$ is a local maximum of $f(x)$ (ii) $f''(c)>0 \Rightarrow f(c)$ is a local minimum of $f(x)$.

Index Numbers and Time Based Series

INDEX NUMBERS

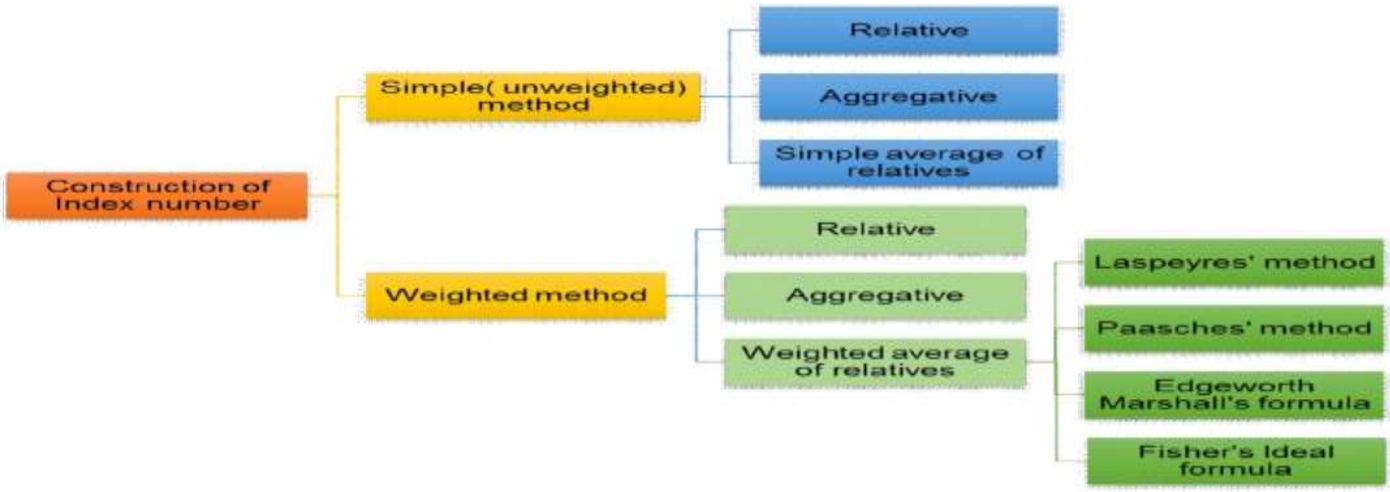
Definition: It is a specialized average designed to measure the changes in the level of an activity or item such as Economy, Price, Value, Trend, Inflation, Purchasing power etc, either with respect to time or geographic location or some other characteristics. It is described either as a ratio or percentage.

Examples: Wholesale Price Index, Consumer Price Index

Classification

i. Price Indexes ii. Quantity Indexes iii. Value indexes iv. Special purpose indexes

Methods of Construction



UNWEIGHTED (SIMPLE) INDEX NUMBERS

Simple (Price) Relative method: It is the price of a certain item in current year to the same in base year expressed as a percentage; that is

Index number $P_{01} = \frac{p_1}{p_0} X100$; where p_1 and p_0 are current year price and base year price respectively.

Simple Aggregative method: In this method the sum of prices of all items under consideration in current year is divided by sum of prices of all the items in base year and multiplied by 100. That is

Index number $P_{01} = \frac{\sum p_1}{\sum p_0} X 100$

Simple Average of Price Relatives method: In this method, we first calculate the price relative of each item and then take their average to calculate the index number, that is

$P_{01} = \frac{\sum\left(\frac{p_1}{p_0} X\ 100\right)}{N}$; Where N is the number of items.

WEIGHTED INDEX NUMBERS

When all items are not of equal importance, we assign weights to each item reflecting their relative importance. Weights are assigned directly or the quantities of items can be used as weights.

Weighted (Price) Relative method: It is the weighted price of a certain item in current year to the same in base year expressed as a percentage; that is

Index number $P_{01} = \frac{p_1 w}{p_0 w} X 100$; where p_1w and p_0w are weighted current year price and weighted base year price respectively.

Weighted Aggregative method: Index number $P_{01} = \frac{\sum p_1 w}{\sum p_0 w}\ X\ 100$

a. **Laspeyre's Index Number:** In this method, the quantities of the base year (q_0) are taken as weights.
   Index number $P_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0}\ X\ 100$
b. **Paasche's Index Number:** In this method, the quantities of current year (q_1) are taken as weights.
   Index number $P_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_1}\ X\ 100$
c. **Edgeworth Marshall Index Number:** In this method, index number is calculated by taking the average of quantities of the base year and the current year.
   Index number $P_{01} = \frac{\sum p_1 (q_0 + q_1)}{\sum p_0 (q_0 + q_1)}\ X\ 100$
d. **Fisher's Ideal Index Number:** It is equal to the geometric mean of Laspeyre's and Paasche's index numbers. Index number $P_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0}\ X\ \frac{\sum p_1 q_1}{\sum p_0 q_1}}\ X\ 100$

Weighted Average of Price Relatives method:

Index number $P_{01} = \frac{\sum\left(\frac{p_1}{p_0} X\ 100\right) X\ w}{\sum w} = \frac{\sum Iw}{\sum w}$

where $I = \frac{p_1}{p_0}\ X\ 100$, the price relative.

Test of Adequacy of Index Numbers

There are 4 types of tests:

i. Unit Test
ii. Time Reversal Test
iii. Factor Reversal Test
iv. Circular Test

We have only Time Reversal Test in the syllabus

Time Reversal Test: If the product of the index number of current year computed on the basis of the base year (P_{01}) and the index number of base year computed on the basis of current year (P_{10}) is equal to 1, that is P_{01} X P_{10} = 1; we say that the method used to calculate the index number works both ways; forward and backward, in respect of time.

Time Reversal Test is satisfied by both Simple Aggregative method and Fisher's method but not by any other.

In Simple Aggregative method $P_{01} = \frac{\sum p_1}{\sum p_0}$ and $P_{10} = \frac{\sum p_0}{\sum p_1}$

=> P_{01} X P_{10} = 1

In Fisher's method $P_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} X \frac{\sum p_1 q_1}{\sum p_0 q_1}}$

and $P_{10} = \sqrt{\frac{\sum p_0 q_0}{\sum p_1 q_0} X \frac{\sum p_0 q_1}{\sum p_1 q_1}}$

=> P_{01} X P_{10} = 1

Sample Question Paper-1

CLASS: XII

Session: 2021-22

Applied Mathematics (Code 241)

Term - 1

Time Allowed: 90 minutes **Maximum Marks: 40**

General Instructions:

1. **This question paper contains three sections – A, B and C. Each part is compulsory.**
2. **Section - A has 20 MCQs, attempt any 16 out of 20.**
3. **Section - B has 20 MCQs, attempt any 16 out of 20**
4. **Section - C has 10 MCQs, attempt any 8 out of 10.**
5. **There is no internal choice in any section.**
6. **All Questions carry equal Marks.**

SECTION – A

In this section, attempt any 16 questions out of Questions 1 – 20.
Each Question is of 1 mark weightage

1	The remainder when 16^{53} is divided by 7 is				
	(a) 1	(b) 2	(c) 3	(d) 4	1
2	Given that x, y and b are real numbers and $x < y,\ b < 0,$ then				
	(a) $\frac{x}{b} < \frac{y}{b}$	(b) $\frac{x}{b} \leq \frac{y}{b}$	(c) $\frac{x}{b} > \frac{y}{b}$	(d) $\frac{x}{b} \geq \frac{y}{b}$	1
3	A man can row at 10 km/hr in still water. If the river is running at 2 km/hr, it takes him 75 minutes to row to a place and back. How far is the place				
	(a) 4.5 km	(b) 5 km	(c) 6 km	(d) 8 km	1
4	The last two digits of the product 4321 x 3215 are				
	(a) 15	(b) 25	(c) 35	(d) 45	1
5	If A is a square matrix such that $A^T\ A = I,$ then the value of $\|A\|$ is				
	(a) 1	(b) -1	(c) ± 1	(d) 0	1
6	The Cofactor of 7 in the matrix B = $\begin{bmatrix} 2 & 3 & 0 \\ -3 & 1 & 7 \\ 1 & -2 & 5 \end{bmatrix}$ is				
	(a) - 1	(b) - 7	(c) 1	(d) 7	1
7	If A and B are square matrices of the same order 3, such that $\|A\| = 4\ and\ AB = 4I.$ Then the				

	value of $\|B\|$ is				
	(a) 4	(b) 16	(c) 32	(d) 64	1
8	The positive value of x which makes the following pair of determinants equal is $\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}, \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$				
	(a) 2	(b) 4	(c) 8	(d) -4 and 4	1
9	The total revenue in Rupees received from the sale of x units of a product is given by R(x) = $3x^2 + 36x + 5$. The marginal revenue, when x = 15 is				
	(a) 116	(b) 96	(c) 90	(d) 126	1
10	The function $y = x^2-10x+7$ is strictly increasing in the interval(s)				
	(a) $(0,5)$ only	(b) $(-\infty,0)$ only	(c) $(5,\infty)$	(d) **R**	1
11	The point on the curve $y=12x-x^2$ where the slope of the tangent is zero.				
	(a) (0,0)	(b) (2,16)	(c) (3,9)	(d) (6,36)	1
12	A candidate claims 30% of the people in her constituency would not vote for her. If 120000 valid votes are polled, then the number of votes she expects from her constituency is				
	(a) 100000	(b) 84000	(c) 56000	(d) 36000	1
13	The range of normal distribution is				
	(a $0\ to\ 1$	(b) $0\ to\ \infty$	(c) $-1\ to\ 1$	(d) $-\infty\ to\ \infty$	1
14	In normal curve, the ordinate is highest at				
	(a) mean	(b) Variance	(c) Standard Deviation	(d) Not fixed	1
15	Consider a Poisson distribution for the tossing of a biased coin. The mean for this distribution is μ. The standard deviation for this distribution is given by				
	(a) $\sqrt{\mu}$	(b) μ^2	(c) μ	(d) $\frac{1}{\mu}$	1
16	If X is a Poisson random variant with mean 3, them $P\{\|X-3\|<1\}$ will be				
	(a) $\left(\frac{99}{8}\right) e^{-3}$	(b) $\left(\frac{9}{2}\right) e^{-3}$	(c) $\left(\frac{9}{8}\right) e^{-3}$	(d) $\left(\frac{3}{2}\right) e^{-3}$	1

17 A random variable X has the following probability function

X	0	1	2	3	4	5	6	7
P	0	a	2a	2a	3a	a^2	$2a^2$	$7a^2 + a$

Then $P(X < 3)$ IS

	(a) $\frac{3}{10}$	(b) $\frac{5}{10}$	(c) $\frac{3}{8}$	(d) $\frac{3}{5}$	1

18 Which index number is called as ideal index number

	(a) Laspeyres	(b) Paasches	(c) Fisher	(d) All of these	1

19 A manufacturer purchases four distinct raw materials, that differ in unit price as given below:

COMMODITY	**UNIT PRICE (Rs) Year 2000**	**UNIT PRICE (Rs) Year 2008**
A	3.20	3.8
B	1.70	2.1
C	148.10	149.50
D	34	45

An unweighted aggregate price index for year 2008 using year 2000 as the base period is

	(a) 97	(b) 107	(c) 117	(d) 127	1

20 The best average for constructing an index number is

	(a) AM	(b) GM	(c) HM	(d) All of these	1

SECTION – B

In this section, attempt any 16 questions out of the Questions 21- 40.
Each Question is of 1-mark weightage.

21 In a 500 metre race, Sophia beats Avni by 20 metres or 4 seconds. Then Sophia's time over the course is

	(a) 25 seconds	(b) 96 seconds	(c) 100 seconds	(d) 104 seconds	1

22	It is 7:00 P.M. currently. What time (in A.M. or P.M.) will be in next 1500 hours?				
	(a) 7:00 PM	(b) 9.10 AM	(c) 7:00 AM	(d) 10.40 PM	1
23	A, B and C hire a shed for Rs2190. A put in 10 cows for 20 days; B 30 cows for 8 days and C 16 cows for 9 days. Then the rent of C is				
	(a) Rs 540	(b) Rs750	(c) Rs900	(d) Rs450	1
24	If Ƞ (n) = Sum of all positive divisors of n, n ϵ N, then Ƞ(35) is equal to				
	(a) 38	(a) 48	(a) 35	(a) 1225	1
25	In an examination out of 500 students, 70% boys and 80% girls are passed. If total pass percentage was 76%, then the number of girls are				
	(a) 200	(b) 236	(c) 284	(d) 300	1
26	If A and B are symmetric matrices of same order, then what can be said for matrix AB – BA				
	(a) Symmetric matrix	(b) Skew Symmetric matrix	(c) Null Matrix	(d) Unit Matrix	1
27	If matrix $A=\begin{bmatrix} a & 3 & 2 \\ -3 & b & 1 \\ -2 & -1 & c \end{bmatrix}$ is skew symmetric, then value of $a+b+c$ is				
	(a) 1	(b) -1	(c) 0	(d) 2	1
28	Find $\frac{dy}{dx}$, if $x = at^2$, $y = 2at$				
	(a) $\frac{1}{t}$	(b) $\frac{1}{t^2}$	(c) $\frac{1}{t^3}$	(d) $\frac{1}{t^4}$	1
29	The price per unit of a commodity produced by a company is given by p= 30 -2x and 'x' is the quantity demanded. Find the revenue function R, the marginal revenue when 5 commodities are in demand (or produced).				
	(a) 10	(b) 20	(c) 30	(d) 40	1
30	**The equations of the normals to the curve $3x^2 - y^2 = 8$, parallel to the line $x + 3y = 4$ are**				
	(a) $x - 3y - 8 = 0$ and $x + 3y + 8 = 0$		(b) $x + 3y - 8 = 0$ and $x + 3y + 8 = 0$		

	(c) $x + 3y - 8 = 0$ and $-x + 3y + 8 = 0$		(d) None of these		1
31	A salesman wants to know the average number of units he sells per sales call. He check past sales records and comes up with the following probabilities Sales (in units) | 0 | 1 | 2 | 3 | 4 | 5 Probability | 0.15 | 0.20 | 0.10 | 0.05 | 0.30 | 0.20 His expected value of the number of units he sells per sale call is				
	(a) 1.00	(b) 2.25	(c) 2.50	(d) 2.75	1
32	Jobs arrive at a factory at an average rate of 5 in an 8 hours shift. The arrival of the jobs following Poissions distribution. The average service time of job on the factory is 40 minutes. The service time follows exponential distribution. Ideal time (in hours) at the factory per shift will be				
	(a) $\frac{5}{7}$	(b) $\frac{14}{3}$	(c) $\frac{7}{5}$	(d) $\frac{10}{3}$	1
33	A traffic engineer records the number of bicycle riders that use a particular cycle track. He records that an average of 3.2 bicycle riders use the cycle track every hour. Given that the number of bicycles that use the cycle track follow a Poisson distribution, Then the probability that 2 or less bicycle riders will use the cycle track within an hour is				
	(a) 0.21	(b) 0.54	(c) 0.171	(d) 0.381	1
34	A die is thrown again and again until three 5 s' are obtained. Find the probability of obtaining the third 5 in the seventh throw of the die.				
	(a) $C_3^6 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^3 \frac{1}{6}$	(b) $C_2^6 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^4 \frac{1}{6}$	(c) $C_3^6 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^4 \frac{1}{6}$	(d) $C_2^6 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^3 \frac{1}{6}$	1
35	Given that the scores of a set of candidates on an IQ test are normally distributed. If the IQ test has a mean of 100 and a standard deviation of 10, Then the probability that a candidate who takes the test will score between 90 and 110 is				
	(a) 0.8413	(b) 0.1587	(c) 0.3658	(d) 0.6826	1
36	During a certain period, the cost of living index number goes from 110 to 200 and the salary of a worker is also raised from 325 to 500. Does the worker really gains or loses, and by how much amount in real terms?				
	(a) worker actually loses = Rs45·45 in	(b) worker actually gains = Rs45·45 in real	(c) worker actually gains = Rs50 in real	(d) worker actually gains =	1

	real terms.	terms.	terms	Rs90 in real terms	
37	Which of the following measure the general changes in the price level from one period to another				
	(a) Value Index Numbers	(b) Quantity Index Numbers	(c) Price Index Numbers	(d) None of above	1
38	The price index using weighted average of relative method for the food consumption in a student hostel in a month taking data of year 1997 as base year for calculations is				

COMMODITY	**WEIGHT**	**PRICE PER UNIT**	
		1997	2001
Rice	14 quintals	90	120
Wheat	20 kg	30	46
Pulses	35 kg	22	34
Milk	15 litres	50	90

	(a) 152.7	(b) 172.7	(c) 167.2	(d) 176.4	1
39	The whole sale price index of rice in 2012 compared to 2010 is 130. If the cost of rice was Rs22 per kg in 2010 calculate the cost in 2012				
	(a) Rs28.60	(b) Rs 30.45	(c) Rs25.50	(d) Rs27.80	1
40	The Time reversal test is satisfied by				
	(a) Laspeyres index only	(b) Paasches index only	(c) Both Laspeyres and Paasches index numbers	(d) Fishers ideal index	1
	SECTION – C **In this section, attempt any 8 questions out 10 Questions.** **Each question is of 1 mark weightage. (Questions 46-50 are based on a Case-Study).**				
41	Lavanya plans to spend less than Rs 5000 on an electric dryer, including the 9% GST and a				

	Rs 640 setup charge. Then the range of selling price (without GST) in which she can afford the dryer is				
	(a) Less than Rs 4000	(b) Less than Rs 4360	(c) More than Rs 4000	(d) More than Rs 4360	1
42	In 1 minute $\frac{4}{7}$ of a bucket is filled. Then the time in which the rest of bucket can be filled is				
	(a) $\frac{4}{7}$ $minute$	(b) $\frac{3}{7}$ $minute$	(c) $\frac{7}{4}$ $minute$	(d) $\frac{3}{4}$ $minute$	1
43	A toy manufacturing firm assesses its variable cost to be 'x' times the sum of 30 and 'x', where 'x' is the number of toys produced, also the cost incurred on storage is Rs 1500. Find the total cost function and the marginal cost when 20 toys are produced.				
	(a) 60	(b) 70	(c) 80	(d) 90	1
44	A particular river near a small-town floods and overflows twice in every 10-years on an average. Assuming that the Poisson distribution is appropriate, what is the mean expectation. Also calculate the probability of 3 or less overflow floods in a 10-year interval.				
	(a) 0.18	(b) 0.86	(c) 0.54	(d) 0.72	1
45	In a district, exam scores of 300 student of class XII are recorded at the end of the session. Ramesh scored 800 marks in total out of 1000. The average score for the batch was 700 and the standard deviation was calculated to be 180. Then the score of Ramesh in compare to his batch mates in the whole district was				
	(a) Ramesh did better than 65.82% of students in the district.				
	(b) Ramesh did better than 67.68% of students in the district.				
	(c) Ramesh did better than 71.23% of students in the district.				
	(d) Ramesh did better than 76.43% of students in the district.				1
	CASE STUDY A factory produces three items every day. Their production on certain day is 45 Tons. It is found that the production of third item exceeds the production of first item by 8 tons while the total production of first and third item is twice the production of second item.				

	Answer the following question.				
46	If x, y, z respectively denote the quantity (in tons) of first, second and third item produced, then which of the following is true?				
	(a) $x + y + z = 45$	(b) $x + 8 = z$	(c) $x - 2y + z = 0$	(d) All of these	1
47	If $\begin{bmatrix}1 & 1 & 1\\1 & 0 & -2\\1 & -1 & 1\end{bmatrix}^{-1} = \frac{1}{6}\begin{bmatrix}2 & 2 & 2\\3 & 0 & -3\\1 & -2 & 1\end{bmatrix}$ Then the inverse of $\begin{bmatrix}1 & 1 & 1\\1 & 0 & -1\\1 & -2 & 1\end{bmatrix}$ is				
	(a) $\begin{bmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\\frac{1}{2} & 0 & \frac{-1}{2}\\\frac{1}{6} & \frac{-1}{3} & \frac{1}{6}\end{bmatrix}$	(b) $\begin{bmatrix}\frac{1}{3} & \frac{1}{2} & \frac{1}{6}\\\frac{1}{3} & 0 & \frac{-1}{3}\\\frac{1}{6} & \frac{-1}{3} & \frac{1}{6}\end{bmatrix}$	(c) $\begin{bmatrix}\frac{1}{2} & 0 & \frac{-1}{2}\\\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\\frac{1}{6} & \frac{-1}{3} & \frac{1}{6}\end{bmatrix}$	(d) None of these	1
48	$x : y : z$ *is equal to*				
	(a) 12 : 13 : 20	(b) 11: 15 : 19	(c) 15 : 19: 11	(d) 13 : 12 : 20	1
49	Which of the following is true				
	(a) $\lvert adj\ A\rvert = \lvert A\rvert^{n-1}$,where n is order of the matrix A				
	(b) $(A')^{-1} = (A^{-1})'$ =A is skew symmetric matrix of odd then $\lvert A\rvert = 0$				
	(c) A is skew symmetric matrix of odd then $\lvert A\rvert = 0$				
	(d) $If\ A = \left[a_{ij}\right]_{3\times3}\ \ then\ \lvert K.A\rvert = K^3\lvert A\rvert$				1
50	Which of the following is true in the given determinant of A, where $A = \left[a_{ij}\right]_{3\times3}$				
	(a) Order of minor is less than order of the det (A)				
	(b) Value if a determinant is obtained by multiplying elements of a row or column by corresponding cofactors.				
	(c) Order of minors and cofactors of same elements of A is same.				1
	(d) All of these.				

Class : XII (Applied Mathematics)
Session: 2021-22

QN	Correct Option	Hints / Solution
1	d	$16^{53} \equiv 2^{53} \pmod 7 \equiv 8^{17}\, 2^2 \pmod 7 \equiv 4 \pmod 7$
2	c	$\frac{x}{b} > \frac{y}{b}$
3	c	Speed downstream = 12 km /hr and Speed upstream = 8 km /hr $\frac{x}{12} + \frac{x}{8} = \frac{75}{60} \Rightarrow$ x = 6 km
4	a	(4321 x 3215) mod 100 = (21 x 15) mod 100 = 315 mod 100 = 15
5	c	$\lvert A^T A\rvert = \lvert I\rvert \Rightarrow \lvert A\rvert^2 = 1 \Rightarrow \lvert A\rvert = \pm 1$
6	d	$C_{23} = -\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = 7$
7	b	$\lvert AB\rvert = \lvert 4I\rvert = 4^3\lvert I\rvert = 64$ $\lvert AB\rvert = \lvert A\rvert\lvert B\rvert \Rightarrow 64 = 4\lvert B\rvert \Rightarrow \lvert B\rvert = 16$
8	b	$2x^2 - 15 = 32 - 15 \Rightarrow x = \pm 4 \ but\ x > 0. \Rightarrow x = 4$
9	d	126
10	c	(5,∞)
11	d	(6,36)
12	b	84000
13	d	$(-\infty\ to\ \infty)$
14	a	Mean
15	a	$\sqrt{\mu}$
16	b	$\left(\frac{9}{2}\right) e^{-3}$
17	a	Paasche's' index
18	c	Fisher
19	d	100
20	b	GM
21	b	Time taken by Avni to cover the course is = $\frac{4}{20} \times 500 = 100\ seconds$

<table>
<tr><td></td><td></td><td>Time taken by Sophia over Avni is = 100 – 4 = 96 seconds</td></tr>
<tr><td>22</td><td>c</td><td>(2+45)mod7 = 5 mod 7 i.e. Friday</td></tr>
<tr><td>23</td><td>a</td><td>Ratio of rent to be paid by A,B,C = 10 x 20 : 30 x 8 : 16 x 9 = 25 : 30 : 18
C's share of rent = $\frac{18}{73} \times 2190 = Rs\ 540$</td></tr>
<tr><td>24</td><td>c</td><td>$\eta(35) = 1 + 5 + 7 + 35 = 48$</td></tr>
<tr><td>25</td><td>d</td><td>Ratio of boys and girls = 2:3 $\Rightarrow$ Number of girls = $\frac{3}{5} \times 500 = 300$</td></tr>
<tr><td>26</td><td>b</td><td>$(AB - BA)^c = -(AB - BA)$</td></tr>
<tr><td>27</td><td>c</td><td>a = 0 , b = 0 , c = 0 so a + b + c = 0</td></tr>
<tr><td>28</td><td>a</td><td>$\frac{1}{t}$</td></tr>
<tr><td>29</td><td>a</td><td>Revenue function is, R= px= $(30x - 2x^2)$
MR= 30-4x
MR when 5 commodities are in demand is Rs 10.</td></tr>
<tr><td>30</td><td>b</td><td>---</td></tr>
<tr><td>31</td><td>b</td><td>$f'(x) = x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3)$
$f'(x) = 0 \Rightarrow x = 1,2,3$
<table>
<tr><th>interval</th><th>Sign of f' (x)</th><th>Nature of f(x)</th></tr>
<tr><td>$(-\infty, 1)$</td><td>-</td><td>Decreasing</td></tr>
<tr><td>(1,2)</td><td>+</td><td>Increasing</td></tr>
<tr><td>(1,2)</td><td>-</td><td>Decreasing</td></tr>
<tr><td>$(3, \infty)$</td><td>+</td><td>Increasing</td></tr>
</table></td></tr>
<tr><td>32</td><td>b</td><td>$\lambda = \frac{5}{8},\ \mu = \frac{3}{2}\quad \rho = \frac{5}{12}$
Ideal time = $1 - \rho = \frac{7}{12}$
Therefore, idle time for 8- hour shift = $\frac{7}{12} \times 8 = \frac{14}{3}\ hours$</td></tr>
<tr><td>33</td><td>d</td><td>
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>P</td><td>0.15</td><td>0.20</td><td>0.10</td><td>0.05</td><td>0.30</td><td>0.20</td></tr>
<tr><td>XP</td><td>0</td><td>0.20</td><td>0.20</td><td>0.15</td><td>1.20</td><td>1.00</td></tr>
</table></td></tr>
</table>

		$E(X) = \ 0.20 + 0.20 + 0.15 + 1.20 + 1.00 = \ 2.75$
34	b	$P(third\ 5\ on\ seventh\ throw\ of\ die\ )$ $= \ P\ (\ two\ 5s' onsix\ throws) X\ P(a\ 5\ on\ the\ next\ single\ thrown\ of\ die)$ $= C_2^6 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^4 \ \frac{1}{6}$
35	d	P(90 < X < 110) = P(X < 110) - P(X < 90) P(90 < X < 110) = P (–1 < Z < 1) = P (Z < 1) – P(Z < –1) = 0.8413 – 0.1587 = 0.6826
36	b	The probability of obtaining two head and one tail is = $\frac{3}{8}$
37	d	$E(X) = \ Var(X) = \ \lambda = 3.2$ Therefore $P(X \leq 2) = \ P(X = 0) + \ P(X = 1) + P(X = 2) = 0.041 + 0.13 + 0.21 = 0.381$
38	a	Weighted price = $I_{2001} = \frac{5140}{3380} \times 100 = \ 152.07$
39	b	Therefore, the index of the year 2008 on the base year 2000 = $I_{2000} = \frac{200.4}{187} \times 100$ $= \ 107.165 \ = 107.2$
40	a	
41	a	$x + 0.09x \ 640 < 5000 \ \Rightarrow \ x < 4000$
42	d	$Since, \frac{4}{7}\ part\ of\ bucket\ is\ filled\ in\ 1\ minute\ .$ $therefore\ 1\ part\ of\ bucket\ is\ filled\ in\ \frac{7}{4}\ minute$ $1 - \frac{4}{7} = \ \frac{3}{7}\ part\ of\ bucket\ is\ filled\ in \frac{7}{4} \times \frac{3}{7} = \frac{3}{4}\ minute$
43	b	The total cost function C(x) is given by, C(x) = x (x + 30) + 1500 = x^2 + 30x + 1500 The marginal cost MC is given by, MC= Dc/dx= 2x+30 Marginal cost of producing 20 toys is MC (20) = dC /dx]$_{x=20}$ = 2(20)+ 30 = 70
44	b	As the average event of flood overflow, in every 50-years is two In the given Poisson distribution, = 2 The goal is to find P(X 3) As

		P (X = 0) = = 0.14 P (X = 1) = = 0.27 P (X = 2) = = 0.27 P (X = 3) = = 0.18 Therefore, P(X 3) = P(X = 0) + P(X = 1) + P(X = 2) +P(X = 3) = 0.14+0.27+0.27+0.18 = 0.86
45		Firstly, we need to find Ramesh's Z-Score and use the respective z-table before we determine how well he has performed as compared to his batch mates As $\mu = 700 \ and \ \sigma = \ 180 \ \ and \ x \ = 800 \qquad z = \frac{800-700}{180} \ = \ 0.56$ Once you have the Z-Score, the next step is choosing between the two Z- tables. In the Z-table, go vertically down on the leftmost column to find the value of the first two digits of your Z Score (0.5 in this case) and then go alongside on the topmost row to find the value of the digits at the second decimal position (.06 in this case). Once you have mapped these two values, the intersection of the row of the first two digits and column of the second decimal point in the table gives the value 0.7123 i.e. the area on the left of ordinate corresponding to Z = 0.56. This area also represents the probability of scoring < 800 marks. Lastly, to get this as a percentage we multiply that number with 100 i.e. 0.7123 x 100 = 71.23%. Hence, we can say that Ramesh did better than 71.23% of students in the district
46	d	According to the question $x + y + z = 45 \quad \ldots i \qquad x + 8 = z \quad \ldots ii$ $x + z = 2y \quad ie \quad x - 2y + z \ = 0 \quad \ldots iii$
47	b	$\begin{bmatrix} \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{3} & 0 & \frac{-1}{3} \\ \frac{1}{6} & \frac{-1}{3} & \frac{1}{6} \end{bmatrix}$
48	b	$X = \ (B')^{-1} \, C \ = \begin{bmatrix} \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{3} & 0 & \frac{-1}{3} \\ \frac{1}{6} & \frac{-1}{3} & \frac{1}{6} \end{bmatrix} \begin{bmatrix} 45 \\ -8 \\ 0 \end{bmatrix} = \begin{bmatrix} 11 \\ 15 \\ 19 \end{bmatrix}$
49	d	$If \ A = \ [a_{ij}]_{3\times3} \quad then \ \ |K.A| = K^3|A|$

50	d	All of these.

Sample Question Paper -2

Term- 1: Session 2021-22

Class- XII

Subject- Applied Mathematics (241)

Time Allowed: -90 minutes **Maximum marks: -40**

General Instructions:

1. This question paper contains three sections – A, B and C. Each part is compulsory.
2. Section - A has 20 MCQs, attempt any 16 out of 20.
3. Section - B has 20 MCQs, attempt any 16 out of 20
4. Section - C has 10 MCQs, attempt any 8 out of 10.
5. There is no internal choice in any section.
6. All Questions carry equal Marks.

SECTION – A

In this section, attempt any 16 questions out of Questions 1 – 20. Each Question is of 1-mark weightage.

1	[(3 × 7) + 5] (mod 4) is (a) 4 (b) 2 (c) 5 (d) 3	1
2	If x ∈ R, |x| < 3, then (a) -3 < x < 3 (b) 3 < x < -3 (c) x = ϕ (d) -3 < x ≤ 3	1
3	It is currently 8:00 A.M. What time (in A.M. or P.M) will be in next 500 hours? (a) 4:00 A.M. (b) 2:00 P.M. (c) 4:00 P.M. (d) 3:00 P.M.	1
4	A pipe can fill a tank in 40 minutes. Due to a leakage in the bottom it took 60 minutes to fill the tank. How much time will it take for the leakage to empty the full tank? (a) half an hour (b) 1 hour (c) 2 hours (d) 4 hours	1
5	If A=$\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$,then A+$A^T$=I , then the value of α is (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{3\pi}{2}$ (d) π	1
6	The matrix $\begin{bmatrix} 5 & -10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$ is a singular matrix, if the value of b is	1

	(a) -3 (b) 3 (c) 0 (d) does not exist	
7	If A is a square matrix of order 3 and $\|A\| = -2$, then $\|\text{adj}(adj.A)\|$ is equal to (a) -8 (b) 0 (c) 16 (d) 4	1
8	If solving a system of linear equations in 3 variables by Cramer's rule, we get $\Delta = 0$ and at least one of Δx, Δy, Δz is non-zero then the system of linear equations has (a) no solution (b) unique solution (c) infinitely many solutions (d) trivial solution	1
9	If the function $f(x) = k x^3 - 9x^2 + 9x + 3$ is increasing in every interval, then: (a) $k > 3$ (b) $k < 3$ (c) $k > 3$ (d) $k \leq 3$	1
10	The function $f(x)=x^2 - 2x$ is decreasing in the interval (a) $(1,\infty)$ (b) $(-\infty,1)$ (c) $(-1, \infty)$ (d) None of these	1
11	The equation of the normal of the curve y=sin x at (0,0) is (a)y=0 (b)x=0 (c) x + y=0 (d) x - y=0	1
12	A, B and C invested ₹45,000, ₹70,000 and ₹90,000 respectively to start a business. At the end of2 years, they earned a profit of ₹164,000. B's share in the profit is (a) ₹ 64,000 (b) ₹ 36,000 (c) ₹ 56,000 (d) ₹ 72,000	1
13	If the mean and the variance of a probability distribution are 4 and 2 respectively, then the probability of two successes is (a) $\frac{1}{2}$ (b) $\frac{7}{64}$ (c) $\frac{219}{256}$ (d) $\frac{37}{256}$	1
14	A dice is thrown twice, the probability of occurring of 5 at least once is (a) $\frac{11}{36}$ (b) $\frac{7}{12}$ (c) $\frac{35}{36}$ (d) none of these	1
15	If X is a Poisson variable such that P(X=1)=2P(X=2), then P(X=0) is (a) e (b) e^{-1} (c) 1 (d) e^2	1
16	Poisson distribution is a limiting case of	1

	(a)Binomial distribution (b) Normal distribution (c)Both (d)None	
17	Find the variance of probability distribution of number of sixes in three throws of a die. (a) $\frac{5}{12}$ (b) $\frac{1}{2}$ (c) $\frac{7}{12}$ (d) None of these	1
18	The wholesale price index (or price relative) of rice in 2018 compared to 2015 is 150. If the costof rice was ₹24 per kg in 2015, calculate the cost in 2018. (a) ₹ 64 (b) ₹ 72 (c) ₹ 56 (d) ₹ 36	1
19	If $\frac{\lvert x+1\rvert}{x+1} > 0$, x ∈ R, then (a) x ∈ [- 1, ∞) (b) x ∈ (- 1, ∞) (c) x ∈ (- ∞, -1) (d) x ∈ (- ∞, - 1)	1
20	In what ratio must water be mixed with milk to gain $16\frac{2}{3}$ % on selling the mixture at cost price? a) 1:6 b) 4:3 c) 6:1 d) 2:3	1

SECTION – B

In this section, attempt any 16 questions out of the Questions 21 - 40. Each Question is of 1-mark weightage.

21	A boat covers a certain distance downstream in 1 hour while it comes back in $1\frac{1}{2}$ hours. If the speed of the stream be 3 km/h, what is the speed of boat in still water? (a) 12 km/h (b) 13 km/h (c) 14 km/h (d) 15km/h	1
22	A can run 22.5 meter while B runs 25 meter in the same time. In a 1000 m race. How much distance B beats A? (a) 200 meter (b) 120 meter (c)40 meter (d) 100 meter	1
23	The function f(x) = $\frac{x}{2} + \frac{2}{x}$ has local minimum at (a) x = 1 (b) x = 2 (c) x = -2 (d) x = -1	1
24	If A is square matrix, then A is symmetric, if (a) A^2=A (b)) A^2=I (c) A^T=A (d) A^T=-A	1
25	If A= $\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and kA= $\begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k , a ,b are respectively	1

	(a) -6, -12,-18 (b) -6, 4, 9 (c) -6, -4, -9 (d) - 6, 12, 18	
26	For the system of equations: x+2y+3z=1, 2x+y+3z=2 & 5x+5y+9z=4 (a) There is only one solution (b) there exists infinitely many solution (c) there is no solution (d) none of these.	1
27	If $x = 2at$ and y=t^2 then find $\frac{d^2y}{dx^2}$ (a) $\frac{1}{2a^2}$ (b) $2a^2$ (c) $\frac{1}{2t^2}$ (d) $\frac{t}{2a^2}$	1
28	Find the point of local maxima for f(x) = (x -1)$(x-2)^2$? (a) 2 (b) $\frac{4}{3}$ (c) 0 (d) -2	1
29	The total cost function is given by C(x)= $\frac{1}{3}x^3 - 5x^2 + 30x - 15$ and the selling price per unit is ₹ 6 . Find for what value of x will the profit be maximum. (a) 2 (b) 4 (c) 8 (d) 6	1
30	The value of x, if [x 1] $\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = 0$ (a) -2 (b) 2 (c) -1 (d) 0	
31	Find the value of A^2, where A is 2 x 2 matrix whose elements are given by $a_{ij} = \begin{cases} 1 \; if \; i \neq j \\ 0 \; if \; i = j \end{cases}$ (a) I (b) 2 I (c) 0 (d) - I	1
32	Given that matrices A and B are of order 3 x n and m x 5 respectively, then the order of matrix C = 5 A + 3 B is: (a) 3 x 5 and m = n (b) 3 x 5 (c) 3 x 3 (d) 5 x 5	1
33	Suppose that X has a Poisson distribution. If P(X=2)= $\frac{2}{3}$ P(X=1),evaluate P(X=0). (a)$e^{\frac{4}{3}}$ (b) $e^{\frac{-5}{3}}$ (c) $e^{\frac{-4}{3}}$ (d) none of these	1
34	If $\sum p_0$ = 113 and $\sum p_1$ = 142, then p_{01} is (a) 122.41 (b) 125.66 (c) 129.23 (d) 132.57	1

35	A random variable 'X' has the following probability distribution: X | 0 | 1 | 2 | 3 P(X) | k | K | 2k | k Find the value of P(x=1) + P(x=2)+ P(x=3) (a)1 (b) 0.2 (c) 0.5 (d)0.8	1
36	A box contains 100 bulbs of which 10 are defective. The probability that out of a sample of 5 bulbs drawn one by one with replacement none is defective is (a) $\left(\frac{1}{2}\right)^5$ (b) $\left(\frac{9}{10}\right)^1$ (c) $\left(\frac{9}{10}\right)^5$ (d) $\left(\frac{1}{10}\right)^5$	1
37	The formula $\frac{\sum p_1 q_0}{\sum p_0 q_0}$ x 100 is used to calculate? (a) Laspeyre's index value (b) Paasche's consumer price index (c)Passche's cost of living index (d) Laspeyre's consumer price index	1
38	Price index by Marshall Edge worth method takes (a) $q0$ as weights (b) $q1$ as weights (c) $q0+q12$ as weights (d) $\sqrt{q0q1}$ as weights	1
39	Index number represents: (a) Specialised average in percentage (b) Symmetry representation (c) Coefficient of variation (d) Deviation	1
40	In a binomial distribution, the sum of its mean and variance is 1 & 8. Find the probability of two successes if the event was conducted 5 times. (a) $\frac{128}{625}$ (b) $\frac{64}{625}$ (c) $\frac{64}{125}$ (d)none of these	1

SECTION – C

In this section, attempt any 8 questions out 10 Questions. Each question is of 1-mark weightage. (Questions 46-50 are based on a Case-Study).

41	The wholesale price index or price relative) of rice in 2012 compared to 2010 is 130. If the cost of rice was Rs.12 per kg in 2010. Calculate the cost in 2012. (a) 15.60 per kg. (b) (a) 15 per kg. (c) (a) 15.75 per kg. (d) None of these	1
42	If matrix A = $\begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$, then A^{-1} is equal to	1

	(a) $\begin{bmatrix} 1 & -\frac{1}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$ (b) $\begin{bmatrix} 1 & \frac{1}{3} \\ 0 & -\frac{1}{3} \end{bmatrix}$ (c) $\begin{bmatrix} 0 & \frac{1}{3} \\ 1 & -\frac{1}{3} \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -\frac{1}{3} \\ 1 & 1 \end{bmatrix}$	
43	If the demand function is p(x)=20 $-\frac{x}{2}$, then the marginal revenue when x=10 is (a) ₹ 5 (b) ₹ 150 (c) ₹ 15 (d) ₹ 10	1
44	A car hire firm has two cars, which it hires out day by day. The number of demands for cars on each day is distributed as a Poisson distribution with mean 1.5. Calculate the probabilities of days on which some demand is refused. (Use $e^{-1.5}$=0.2231) (a)0.2231 (b) 0.1913 (c) 0.8087 (d)none of these	1
45	For the two sector economy input-output table is given below:	

Output→ / **Input↓**	Industry I	Industry II	Final demand	Total out put
Industry I	16	20	4	40
Industry II	8	40	32	80

Find the technology matrix.

(a) $\begin{bmatrix} 0.4 & 0.5 \\ 0.25 & 0.2 \end{bmatrix}$ (b) $\begin{bmatrix} 0.4 & 0.2 \\ 0.2 & 0.5 \end{bmatrix}$ (c) $\begin{bmatrix} 0.4 & 0.25 \\ 0.2 & 0.5 \end{bmatrix}$ (d) $\begin{bmatrix} 0.2 & 0.4 \\ 0.5 & 0.5 \end{bmatrix}$

Partnership-Investment of Capital for Unequal Periods:

A, B and C engaged in a business and made investment and withdrawal of capital as follows:

January 1, A put Rs.15000, on May1, A added ₹ 10000 and Withdrew ₹ 8000 on October; B put ₹ 20000 on January 1, withdrew ₹ 5000 on July 1 and added ₹ 2000 on November 1; C put ₹ 15000 on January 1, Withdrew ₹. 8000 on April 1 and added Rs. 20000 on September 1.

At the end of the year B's Share of profit is ₹ 9630.

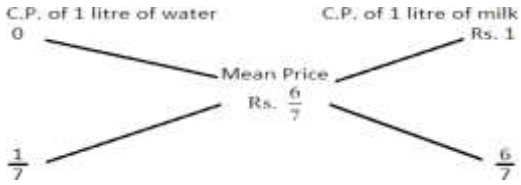
Based on the above information, answer the following questions:

46	A's effective capital (in Rs.) for 1 month is (a) ₹ 57000 (b) ₹ 228000 (c) ₹ 236000 (d) ₹ 328000	1
47	Sum of B's and C's effective capitals (in Rs.) for 1 month is (a) ₹ 107000 (b) ₹ 429000 (c) ₹ 215000 (d) ₹ 214000	1
48	C's share of profit is (a) ₹ 6450 (b) ₹ 7445 (c) ₹ 9675 (d) ₹ 10245	1
49	How much A's share of profit is more than B's share? (a) ₹ 990 (b) ₹ 660 (c) ₹ 720 (d) ₹ 1090	1
50	Sum of A's and C's shares of profit is (a) ₹ 18325 (b) ₹ 20295 (c) ₹ 21495 (d) ₹ 19375	1

Marking Scheme-2
Class XII, TERM - I, 2021-22
Subject – Applied Maths (241)

Q. No.	Correct option	Hints/solution	Marks
1	(b)	[21 + 5] (mod 4) = 21 (mod4) + 5(mod4) = 1 + 1 = 2	1
2	(a)	- 3 < x < 3	1
3	(a)	Find 500(mod24), 500 ≡ 20(mod24) (8 + 20) ≡ 4(mod24) Correct Answer 4:00 A.M.	1
4	(c)	Let the leakage in the tank take x minutes to empty the full tank so, part of the tank emptied by the leakage in 1 minute $=\frac{1}{x}$ Therefore $\frac{1}{40}-\frac{1}{x}=\frac{1}{60}$ $\frac{1}{x}=\frac{1}{40}-\frac{1}{60}=\frac{1}{120}$ ⇨ X=120 = 2 hours	1
5	(b)	A+A^T=I, Equating a_{11}=1 2 cosα = 1	1

		⇨ $\alpha = \frac{\pi}{3}$	
6	(b)	Correct value of b = 3	1
7	(c)	Given $\|A\| = -2$, $\|adj.A\| = \|A\|^2 = 4$ $\|adj(adj.A)\| = 4^2 = 16$	1
8	(a)	If Δ= 0 then system has no solution.	1
9	(a)	f' (x) = 3(kx^2 - 6x + 3) Given: f(x) is increasing in every interval. ⇒ f'(x) > 0 ⇒ 3(kx^2 - 6x + 3) > 0 ⇒ (kx^2 - 6x = 3) > 0⇒ K > 0 and $(-6)^2$ - 4(k)(3) < 0 [∵ ax^2 + bx + c > 0 and D is c < 0] ⇒ k > 0 and $(-6)^2$ - 4(k) (3) < 0 ⇒ k > 0 and 36 - 12k < 0 ⇒ k > 0 and 12k > 36 ⇒ k > 0 and k > 3 Correct Answer ⇒ k > 3	1
10	(b)	f' (x) = 2x – 2 < 0, x ∈ (-∞,1)	1
11	(c)	x + y=0	1
12	(c)	∵ A : B : C = 45000 : 70000 : 90000 A : B : C = 9 : 14 : 18, Total = 9+14+18 =41 ∴ B's share = $\frac{14}{41}$ x 164000 = ₹56000	1
13	(b)	Correct answer = $\frac{7}{64}$ np=4, npq=2,p(2)=${}^{n}_{2}Cp^2q^{n-2}$	1
14	(a)	Req. prob.=1-P(no 5 on both dice) Correct answer = $\frac{11}{36}$	1
15	(b)	e^{-1}, $Hint: \frac{e^{-\lambda}\lambda}{1} = \frac{2}{1}\frac{e^{-\lambda}}{2}\frac{\lambda^2}{}, P(O) = e^{-\lambda}$	1
16	(a)	Binomial distribution	1
17	(a)	$\frac{5}{12}$	1
18	(d)	Let the cost of rice be ₹p_0 and p_1 per kg in 2015 and 2018 respectively ∵p_0 = ₹24 and p_{01} = 150 ∴ $p_{01} = \frac{p_1}{p_0}$ x 100 , 150 = $\frac{p_1}{24}$ x 100 p_1 = 36 Hence, the price of rice in 2018 is ₹36 per kg.	1

19	(b)	$x \in (-1, \infty)$	1
20	(a)	 Let C.P. of 1 litre milk be ₹ 1 , gain = $16\frac{2}{3}$ % The ratio of water and milk = 1/7 : 6/ 7 = 1: 6	1
Q. No.	**Correct option**	**Hints/solution**	**Marks**
21	(d)	Let speed of boat in still water = x km/h Speed of stream = 3 km/h Speed in downstream =(x + 3) km/h Speed in back (upstream) = (x – 3) km/h Distance covered by boat = (x + 3).1 = (x – 3).$\frac{3}{2}$ Speed of boat x = 15 km/h	1
22	(d)	When B run 25 m, A runs = 22.5 m When B runs 1 m then A runs = $\frac{22.5}{25}$ And when B runs 1000 m then A runs $\frac{22.5}{25}$ x 1000 = 900 m Hence B beats A by 100 mtr.	1
23	(b)	f''(x) = $\frac{1}{2} - \frac{1}{x^2}$ = 0, x = -2, 2 f''(x) >0 then f(x) is minimum and f''(2) > 0 correct answer x = 2	1
24	(c)	A^T=A	1
25	(c)	K = -6 , a = -4 and b = -9	1
26	(a)	As $\|A\| \neq 0$, System has only one solution.	1
27	(a)	$\frac{dx}{dt}$ = 2a & $\frac{dy}{dt} = 2t$, $\frac{dy}{dx} = \frac{t}{a}$ Correct answer $\frac{d^2y}{dx^2} = \frac{1}{a}\frac{dt}{dx} = \frac{1}{2a^2}$	1
28	(b)	Correct answer $= \frac{4}{3}$	1
29	(d)	Correct answer = 6 P(x)=R(x)-C(x),where R(x)=px then find x when P(x) maximum	1
30	(b)	x – 2 = 0, x = 2	1
31	(a)	A= $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, A^2 = A.A =I	1

32	(a)	3 x 5 and m = n	1
33	(c)	$e^{\frac{-4}{3}} \frac{e^{-\lambda}\lambda^2}{2} = \frac{2}{3}\frac{e^{-\lambda}\lambda}{1}, P(O) = e^{-\lambda}$	1
34	(b)	$\because p_{01} = \frac{p_1}{p_0} \times 100 = \frac{142}{113} \times 100$ $p_{01} = 125.66$	1
35	(d)	$\because$ k+k+2k+k =1, k =1/5 P(x=1) + P(x=2)+ P(x=3) = k+2k+k = 4k = 4x $\frac{1}{5}$ = 0.8	1
36	(c)	Correct answer $= \left(\frac{9}{10}\right)^5$	1
37	(a)	Laspeyre's index value	1
38	(c)	Marshall- Edgeworth formula uses the arithmetic mean of the base and current year quantities. Therefore $\frac{q_0 + q_1}{2}$ as weights	1
39	(a)	Specialised average in percentage	1
40	(a)	$\frac{128}{625}$ np+npq=1.8,n=5,p=0.2,q=0.8	1
41	(a)	Let the cost of rice be ₹ p per kg in 2012 Given 130 = $\frac{p}{12}$ x 100 , p = ₹15.60 per kg.	1
42	(a)	Correct answer $A^{-1} = \begin{bmatrix} 1 & -\frac{1}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$	1
43	(a)	Marginal revenue M(x) = p(x).x = (20 - $\frac{x}{2}$)x M'(x) at x = 10 = 20 – x = 20 – 10 Correct answer = 10	1
44	(b)	0.1913 λ=mean=1.5 P(more than two cars demanded)=1-P(0)-P(1)-P(2)	1
45	(c)	Technology matrix = $\begin{bmatrix} 0.4 & 0.25 \\ 0.2 & 0.5 \end{bmatrix}$	1
46	(b)	A's effective capital (in Rs.) for 1 month is = ₹ 228000	1
47	(a)	Sum of B's and C's effective capitals (in Rs.) for 1 month is =₹ 107000	1
48	(c)	C's share of profit is = ₹ 9675	1
49	(d)	A's share of profit is more than B's share = ₹ 1090	1
50	(c)	Sum of A's and C's shares of profit is = ₹ 21495	1

SAMPLE QUESTION PAPER-3

CLASS –XII

SESSION -2021-2022

APPLIED MATHEMATICS (241)

TERM -1

TIME ALLOWED : 90MIN MAX MARKS :40

GENRAL INSTRUCTION :

1.**This question paper contain three sections- A,B,C. Each part is compulsory**
2.Section-A has 20 MCQs , attempt any 16 out of 20.
3. Section-B has 20 MCQs , attempt any 16 out of 20.
4. Section-C has 10 MCQs , attempt any 8 out of 10.
5.There is no internal choice in any section.
6.All questions carry equal marks.

SECTION – A

In this section, attempt any 16 questions out of Questions 1 – 20.
Each Question is of 1 mark weightage.

1.	What is the value of $57 \cong 42 \mod 11$ a) 1 b) 2 c) 3 d) 4	1
2.	Milk and water in two vessels A and B are in the ratio 4:3 and 2:3 respectively. In what ratio the liquids in both the vessels should be mixed to obtain a new mixture in vessel C containing half milk and half water? a) 7:5 b) 6:5 C) 8:5 d) 4:5	1
3	A man can row at 5 km/hr in still water. If the river is running at 1 km/hr it takes him 75 minutes to row to a place and back. How far is the place ? a)2.5 km b)3km c)4km d)5km	1
4.	What is the remainder when 2^{200} is divided by 18? a) 4 b) 8 C) 91 d) 111	1
5	If	1

	$\begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$ Then find the values of a, b, c, x, y, and z respectively. a)-2,-7,-1,-3,-5,2 b)2,7,1,3,5,-2 c)1,3,4,2,8,9 d)-1,3,-2,-7,4,5	
6	Total number of possible matrices of order 3 × 3 with each entry o and 1 (a)9 (b)27 (c)81 (d)512	1
7	If A and B are square matrices of order 3 such that $\|A\| = -1$ and $\|B\| = 3$, then find the value of $\|3AB\|$. (a) -9 (b) -27 (C) -81 (d) -343	
	If the equations 2x + 3y + z = 0, 3x + y – 2z = 0 and ax + 2y –bz=0 has non-trivial solution, then (a)a-b=2 (b)a+b+1=0 (c)a+b=3 (d)a-b-8=0	1
9	Find the minor of 6 and cofactor of 4 respectively in the determinant $\Delta=\begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix}$ (a)6,6 (b)6,-6 (c)-6,-6 (d)-6,6	1
10	If f(x) = 1/x , then the interval in which function is decreasing (a) (1,2) (b) (0,∞) (c) (-∞,∞) (d)None of these	1
11	The point on the curves $y = (x - 3)^2$ where the tangent is parallel to the chord joining (3, 0) and (4, 1) is (a) (-7/2,1/4) (b) (5/2,1/4) (c) (-5/2,1/4) (d) (7/2,1/4)	1
12	Find the maximum profit that a company can make, if the profit function is given by $P(x) = 41 + 24x - 18x^2$. (a)25 (b)43 (c)62 (d)49	1
13	A die is thrown twice and sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once? a) 2/5 b) 3/5 c) 5/36 d) 1/18	1

14	A random variable X has the following probability distribution: | X	0	1	2	3	4	5	6	7
---	---	---	---	---	---	---	---	---
P(X)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$ | Find k. a) 1/10 b) 2/5 C) 3/10 d) 2/7	1
15	The mean of a distribution is 60 with standard deviation 5. Assuming that the distribution is normal, what percentage of items be between 65 and 75 a) 15.74 b) 20.74 C) 18.74 d) 19.74	1
16	Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg a) $1-\frac{9^{11}}{10^{10}}$ b) $1-\frac{9^{10}}{10^{11}}$ C) $1-\frac{9^{10}}{10^{10}}$ d) $1-\frac{9^{12}}{10^{10}}$	1
17	If the proportion of defective in a bulk is 4% Find the probability of 2 defective in a sample of 10. It is known that $e^{-.4}$ = .6703. a) 0.536 b) 0.636 C) 0.736 d) 0.836	1
18	To calculate paasche's index number the weights are taken as a) base year prices b) current year prices c) base year quantities d) current year quantities	1
19.	Fisher's ideal index number is a) geometric mean of Laspeyres and Edgeworth Marshall index number b) arithmetic mean of Laspeyre's and Paasche's index number c) harmonic mean of Laspeyre's and Paasche's index number d) geometric mean of Laspeyre's and Paasche's index number	1
20.	The formula to calculate Laspeyre's index number is a) $\frac{\sum p_1q_1}{\sum p_0q_0}$X100 b) $\frac{\sum p_1q_0}{\sum p_0q_1}$X100 *c*) $\frac{\sum p_1q_0}{\sum p_0q_0}$X100 d) $\frac{\sum p_1q_0}{\sum p_1q_1}$X100	1
	SECTION – B ***In this section, attempt any 16 questions out of Questions 21 – 40.*** **Each Question is of 1 mark weightage.**	
21.	To find the Index number by weighted average of price relatives, we use the formula	1

	(a) $\frac{\sum(\frac{p1}{p0})(p_0q_0)}{\sum(p_0q_0)}$ X 100 (b) $\frac{\sum(p_1)(p_0q_0)}{\sum(p_0q_0)}$ X 100 (c) $\frac{\sum(p_0)(p_0q_0)}{\sum(p_0q_0)}$ X 100 (d))$\frac{\sum(\frac{p1}{p0})(p_1q_1)}{\sum(p_1q_1)}$ X 100	
22.	Two athletes Hari and Mohan finish 100m race in 16 sec and 20 sec respectively. By how many meters does Hari defeat Mohan? (a) 25 m (b) 30m (c) 20m (d) 22m	1
23	In which of the technology matrix, Hawkins- Simon conditions are satisfied (a) $\begin{pmatrix}0.2 & 0.9\\0.8 & 0.1\end{pmatrix}$ (b) $\begin{pmatrix}0.7 & 0.3\\0.2 & 1.2\end{pmatrix}$ (c) $\begin{pmatrix}1.02 & 0.5\\0.6 & 0.8\end{pmatrix}$ (d) $\begin{pmatrix}0.3 & 0.2\\0.1 & 0.5\end{pmatrix}$	1
24	The function $y = \|x\|$ is (a) neither differentiable nor continuous at $x = 0$ (b) differentiable and continuous at $x = 0$ (c) continuous but not differentiable at $x = 0$ (d) differentiable but not continuous at $x = 0$	1
25.	Given that x = a cosθ and y = b cosθ , then value of $\frac{d^2y}{dx^2}$ is (a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) 0 (d) 1	1
26	If A and B are square matrices of the same order, then (A+B)(A-B) is (a) A^2-B^2 (b) A^2 –BA-AB -B^2 (c) A^2-B^2+BA - AB (d) A^2 -BA+B^2+AB	1
27	A sales promotion company sells tickets for ₹100 each to win a prize of ₹5 lakhs. If a person buys one of the 10,000 tickets sold, then his expected gain in rupees is (a) -50 (b) 0 (c) 50 (d) 100	1
28	During a pandemic, 20% of the patients who have the disease get complications. If 100 patients of a locality get infected by the disease, then the standard deviation of the number of patient getting complications is: (a) 4 (b) 9 (c) 6 (d) 3	1
29	The variable cost of producing x units is $(x) = x^2 + 2x$. If the company incurs a fixed cost of ₹81,00 then the level of output where the average cost is minimum is (a) 90 units (b) 50 units (c) 100 units (d) 200 units	1

<table>
<tr><td>30</td><td>Suresh, Satish and Sameer agrees to invest for time periods in the ratio 2:3:4. If their profit sharing ratio is 6:7:8 then the ratio of their investments is
(a) 4:5:6 (b) 9:7:6 (c) 8:7:6 (d) 12:21:32</td><td>1</td></tr>
<tr><td>31</td><td>The prices of group of commodities is given in the following table:
<table><tr><td>Commodities</td><td>A</td><td>B</td><td>C</td><td>D</td></tr><tr><td>$p0$ [Price (₹) in 2018]</td><td>50</td><td>25</td><td>110</td><td>108</td></tr><tr><td>$p1$ [Price (₹) in 2019]</td><td>60</td><td>40</td><td>125</td><td>120</td></tr></table>The price index for 2019 taking 2018 as base year using simple aggregative method is:
(a) 80.23% (b) 119.45% (c) 125.5% (d) 128%</td><td>1</td></tr>
<tr><td>32</td><td>The Time reversal test is satisfied by
(a) Laspeyres index only.
(b) Paasches index only
(c) Both Laspeyres and Paasches index numbers
(d) Fishers ideal index</td><td>1</td></tr>
<tr><td>33</td><td>Most widely used weighted index is
(a) Laspeyres index (b) Paasche’s index
(c)Fisher’s ideal index (d) Marshall- Edgeworth index</td><td>1</td></tr>
<tr><td>34</td><td>The index number was first constructed in
(a) 1750 (b) 1760 (c) 1764 (d) 1770
(b)</td><td>1</td></tr>
<tr><td>35</td><td>An electrical supplier distributor has found the daily demand for fluorescent light bulbs
is normally distributed with a mean of 432 and standard deviation of 86. Find the probability that the demand on a particular day exceeds 518 bulbs.
(a)0.1587 (b) 0.3413 (c) 0.7587 (d) 0.8413</td><td>1</td></tr>
<tr><td>36</td><td>Let $m \in Z+$ consider the relation R_m defined as R_m b iff a ≡ b(mod m),then R_m is
(a) reflexive but not symmetric (b) symmetric but not transitive
(c) reflexive, symmetric but not transitive (d) an equivalence relation</td><td>1</td></tr>
<tr><td>37</td><td>If the present time is 8:40 PM, then the time after $876\frac{1}{2}$ hours will be
(a) 8:40 AM (b) 9:10 AM (c) 6:10 PM (d) 10:40 PM</td><td>1</td></tr>
<tr><td>38</td><td>A, B and C enter into a partnership. B contributes $\frac{1}{3}$ rd of the capital, while A contributes as much as B and C together contribute. The ratio of their capitals is</td><td>1</td></tr>
</table>

	(a) 1:2:3 (b) 3:2:1 (c) 3:1:1 (d) 2:1:1	
39	An insurance company has found that 50% of its claims are for damages resulting from accidents. The probability that a random sample of 10 claims will contain fewer than 2 for accidents is (a) $\frac{1}{1024}$ (b) $\frac{5}{512}$ (c) $\frac{15}{1024}$ (d) $\frac{11}{1024}$	1
40	The value of mortgage loans made by a certain bank is normally distributed with mean of ₹36 lacs and a standard deviation of ₹12 lacs. The probability that a randomly selected mortgage loan is less than 54 lacs is (a) 88.23% (b) 93.32% (c) 97.42% (d) 98.04%	1
	SECTION – C ***In this section, attempt any 8 out of Questions 41-50*** **Each Question is of 1 mark weightage.** **Question 46-50 are based on Case study.**	
41	For a poisson distribution model if arrival rate of passengers at an airport is recorded as 30 per hour on a given day. Find the expected number of arrivals in the first ten minutes of an hour. a) 5 b)4 c)3 d) none	1
42	A man rows 15 km upstream and 25 km downstream in 5 hour each time .What is the speed of current? a) 2km/hour b)1km/hour c)3km/hour d)4km/hour	1
43	A container contains 40 litre milk . From this container 4 litre milk was taken out and replaced with water .This process was repeated further two more times. How much milk is there in the container now? a) 31.16 litre b) 29.16 litre c)35.16 litre d)26 litre	1
44	The demand function of a toy is x= 75-3p and its total cost function is TC =100+3x . For maximum profit the value of x is a) 33 b)31 c)29 d)24	1
45	Given that mean of a normal variable X is 12 and standard deviation is 4 . then Z-score of data point 20 is a) 5 b)4 c)2 d)3	1
	CASE STUDY To promote the making of toilets for women, an organization tried to generate awareness through a) house calls b)emails c)announcements. The cost for each mode per attempt is given below: a)Rs 50 b) Rs 20 c)Rs 40 The number of attempts made in the village X , Y ,Z are given below i) ii) iii)	

	X 400 300 100 Y 300 250 75 Z 500 400 150 Also, the chance of making of toilets corresponding to one attempt of given modes is a) 2% b)4% c)20% Based on the above information , answer the following questions:	
46	The cost incurred by the organization on village X is a) Rs 10000 b)Rs 15000 c) Rs 30000 d) Rs 20000	1
47	The cost incurred by the organization on village Y is b) Rs 25000 b)Rs 18000 c) Rs 23000 d) Rs 28000	1
48	The cost incurred by the organization on village Z is c) Rs 19000 b)Rs 39000 c) Rs 45000 d) Rs 50000	1
49	The total number of toilets that can be expected after the promotion in village X is a) 20 b) 30 c)40 d)50	1
50	The total number of toilets that can be expected after the promotion in village Z is b) 26 b) 36 c)46 d)56	1

MARKING SCHEME -3

CLASS: XII

Session: 2021-22

TERM - I

Applied Mathematics (Code-241)

Q. No	Correct option	Hint/Solutions	
1	A	4	1
2	A	Milk in Vessel A = 4/7 (Dearer Value) Milk in Vessel B = 2/5 (Cheaper Value) Milk in Vessel C = 1/2 (Mean Value) Dearer : Cheaper = 1/2 - 2/5 : 4/7 - 1/2 = 1/10 : 1/14 Required ratio is 14 : 10 = 7 : 5	1
3	B	Let the distance to the place be x km. Speed downstream =(5+1) km/hr =6 km/hr Speed upstream =(5−1) km/hr =4 km/hr Given, 6x+4x=6075 hrs	1

		⇒122x+3x=45 ⇒125x=45 ⇒x=3	
4	A	4	1
5	A	For comparing value of a, b, c,x , y are -2,-7,-1,-3,-5,2	1
6	D	2^9	1
7	C	|3AB|=32|AB|=27×|A|×|B| ------ (1) ⇒ We have, |A|=−1 and |B|=3 ⇒ So, |3AB|=27×|A|×|B| [From (1)] ⇒ |3AB|=27×(−1)×(3) ∴ |3AB|=−81	1
8	A	⇒**2**(−b+**4**)−**3**(−**3**b+**2**a)+**1**(**6**−a)=**0**⇒2(−b+4)−3(−3b+2a)+1(6−a)=0 ⇒−**2**b+**8**+**9**b−**6**a+**6**−a=**0**⇒−2b+8+9b−6a+6−a=0 ⇒**7**b−**7**a=−**14**⇒7b−7a=−14 ⇒a−b=**2**	1
9	D	Minor -6, cofactor 6	1
10	C	(-∞,∞)	1
11	D	(7/2,1/4)	1
12	D	49	1
13	A	A= sum of the numbers appearing on two dice is 6 =(1,5),(5,1),(2,4),(4,2),(3,3) n(A)=5 B= number 4 has appeared at least once =(1,4),(4,1),(2,4),(4,2),(3,4),(4,3),(4,4),(4,5),(5,4),(4,6),(6,4) A∩B=(2,4),(4,2) n(A∩B)=2 Required probability = P(B/A)=n(A)n(A∩B)=52	1
14	a	$10k^2$+9k-1=0, (10k-1)(k+1)=0 so k=1/10	1
15	a	P(65≤X≤75)=P(65−60)/5<(X−μ)/σ<(75−60)/5 =P(1<Z<3) =P(Z<3)−P(Z<1)	1

		(=NORM.S.DIST(3,TRUE))−(=NORM.S.DIST(1,TRUE)) ≈ 0.9987−0.8413≈0.1574= 15.74%1	
16	C	Probability of an egg being defectiv1e =10/100=1/10 So, probability of an egg being non-defective=1−0.1=0.9 10 eggs are drawn successively with replacement. So, the probability of getting no defective egg =0.9^{10} Hence, the probability that there is at least one defective egg =1-0.9^{10}	1
17	A	α=4/10=0.4 required probability=($e^{-\alpha}$x α^r)/rı =($e^{-o.4}$x 0.4^2)/2ı=0.5362	1
18	A	base year prices	1
19	D	geometric mean of Laspeyre's and Paasche's index number	1
20	C	$\frac{\sum p_1 q_0}{\sum p_0 q_0}$X100	1
		Section - B	
21	a	$\frac{\sum(\frac{p1}{p0})(p_0 q_0)}{\sum(p_0 q_0)}$ X 100	1
22	c	Since Hari is faster by 4 secs. ∴ he beats Mohan by = $\frac{100}{20}$× 4 = 20 *meters*	1
23	d	∵ $1 - a11, 1 - a22 > 0\ and\ |I - A| > 0$ and it is true for $\begin{pmatrix} 0.3 & 0.2 \\ 0.1 & 0.5 \end{pmatrix}$	1
24	c	$y = |x|$ has a sharp point at $x = 0$ $y = |x|$ is continuous but not differentiable at $x = 0$	1
25	c	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-b\ sin\theta}{-a\ sin\theta} = \frac{b}{a}; \frac{d^2y}{dx^2} = 0$	1
26	c	A^2-B^2+BA - AB	1
27	a	**Prize(x_i)** **p_i** **x_i p_i** 500000 $\frac{1}{10000}$ 50 0 $\frac{9999}{10000}$ 0 So , ∑ **xi pi** = 50 Net expected gain = 50 – 100 = -50 So gain is – 50.	1

28	a	$n = 100, p = 2/10, q = 8/10, \sigma = \sqrt{npq} = \sqrt{100\ X\ \frac{2}{10} X \frac{8}{10}} = 4$	1
29	a	$TC = VC + FC = x^2 + 2x + 8100$ $AC = x+2+\frac{8100}{x}$ $\frac{d(AC)}{dx} = 1 - \frac{8100}{x^2} = 0$ $X^2 = 8100$ $\therefore x = 90$	1
30	b	Time ratio = 2 : 3 : 4 Profit sharing ratio = 6: 7: 8 Investment ratio = $\frac{6}{2} : \frac{7}{3} : \frac{8}{4}$ = 9:7:6	1
31	b	$\frac{\sum p_1}{\sum p_0} X\ 100 = \frac{345}{340} = 119.45$	1
32	d	Fisher's ideal index	1
33	a	Laspeyres index	1
34	c	1764	1
35	a	$P(x > 518) = 1 - p(x < 518)$ $= 1 - P(z < 1) = 1 - 0.8413$ $= 0.1587$	1
36	d	The relation R_m defined as a ≡ b (mod m) is reflexive, symmetric and transitive ∴ R_m is an equivalent relation	1
37	b	∵ 876 (mod24) = 12 ∴ 8.40 PM will change to 8.40 AM after 12 hours, further after 30 minutes the time will be 9.10 AM	1
38	b	Let total capital be = x & let C's contribution = y, B's contribution= $\frac{x}{3}$, A's contribution = $\frac{x}{3} + y$ Now (A+B+C)'s contribution = $x \Rightarrow x = 6y$ *hence their contribtions are* $2y + y : 2y : y$ *i.e.*, *in the ratio* 3: 2: 1	1
39	c	$P(r < 2) = P(0\ or\ 1) = 10C_0 (\frac{1}{2})^{10} = \frac{1+10}{1024} = \frac{11}{1024}$	1
40	b	$P(x < 54) = P(z < 1.5)$ = 0.9332 = 93.32 %	

41	a	E(x)=30x(1/6) =5	1
42	b	Speed of boat is x km/h ,speed of current is y km/h (x+y)5=25 and (x-y)3=15 x = 4 , y=1km/h	1
43	b	Milk = $40\left(1-\frac{4}{40}\right)^3$ =29.16	1
44	a	TR=px=$(75x-x^2)/3$ P=TR-TC=$(75x-x^2)/3$ –(3x-100) $\frac{dp}{dx}$ = 22-2x/3 =0 x=33	1
45	c	Z=(20-12)/4 =2	1
46	C	$\begin{pmatrix}A\\B\\C\end{pmatrix}=\begin{pmatrix}400&300&100\\300&250&75\\500&400&150\end{pmatrix}\begin{pmatrix}50\\20\\40\end{pmatrix}$ cost incurred by the organization on village X =30000	1
47	C	cost incurred by the organization on village Y=23000	1
48	b	cost incurred by the organization on village Z=39000	1
49	C	$\begin{pmatrix}X\\Y\\Z\end{pmatrix}=\begin{pmatrix}400&300&100\\300&250&75\\500&400&150\end{pmatrix}\begin{pmatrix}2/100\\4/100\\20/100\end{pmatrix}$ The total number of toilets that can be expected after the promotion in village X =40	1
50	d	The total number of toilets that can be expected after the promotion in village Z=56	1

Sample Question Paper - 4

CLASS: XII

Session: 2021-22

Applied Mathematics (Code-241)

Term - 1

Time Allowed: 90 minutes **Maximum Marks: 40**

General Instructions:

1. **This question paper contains three sections – A, B and C. Each part is compulsory.**
2. **Section - A has 20 MCQs, attempt any 16 out of 20. 3. Section - B has 20 MCQs, attempt any 16 out of 20**
4. **Section - C has 10 MCQs, attempt any 8 out of 10.**
5. **There is no internal choice in any section.**
6. **All Questions carry equal Marks.**

SECTION – A

In this section, attempt any 16 questions out of Questions 1 – 20.
Each Question is of 1 mark weightage.

1.	The value of 18 x10 (mod 7) is : (a) 5 (b) 4 (c) 3 (d) 2	1
2.	The length of rectangle is double the breadth. If the minimum perimeter of the rectangle is 120 CM, then (a) Breadth>20cm (b) Breadth < 20cm (c) Breadth >= 20cm (d) Breadth <= 20cm	1
3.	A boat running downstream covers a distance of 16 km in 2 hours while for covering the same distance upstream it takes 4 hours. What is the speed of the boat in still water? (a) 4 km/h (b) 6 km/h (c) 8 km/h (d) 10 km/h	1
4.	If $x \equiv 4 \pmod 7$, then positive values for x are : (a) {4,11,18,...} (b) {11,18,25,...} (c) {4,8,12,...} (d) {1,8,15,...}	1
5.	If A is a square matrix of order 3 and $\|A\| = 2$, then the value of $\|-AA'\|$ is : (a) 4 (b) 2 (c) -2 (d) -4	1
6.	If A and B are square matrices of same order such that $\|A\| = -3$ and AB =I, then the value of $\|B\|$ is : (a) 3 (b) 1/3 (c) -3 (d) -1/3	1

7.	If the area of the triangle with vertices (1, -1), (-4, k) and (-3,-5) is 24 square units then the values of k are : (a) -6, 18 (b) 16, 8 (c) 6, -18 (d) -16, 8	1
8.	If solving a system of linear equations in 3 variables by Cramer's rule, we get D ≠ 0 and $D_1 = D_2 = D_3 = 0$, then the system of linear equations has : (a) no solution (b) unique solution (c) infinitely many solutions (d) trivial solution	1
9.	The demand function of a monopolist is given by x = 100 - 4p. The quantity at which MR (marginal revenue) = 0 will be (a) 25 (b) 10 (c) 50 (d) 40	1
10.	The function f(x) = x^x, x > 0, is decreasing on the interval (a) (0, e] (b) (0, 1/e) (c) [1/e, ∞) (d) none of these	1
11.	The maximum slope of the tangent to the curve $y = -x^3 + 3x^2 + 9x - 27$ is (a) 0 (b) 12 (c) 16 (d) 32	1
12.	In what ratio must a grocer mix two variety of pulses costing Rs.85 per kg and Rs.100 per kg respectively so as to get a mixture worth Rs. 92 per kg? (a) 7 : 8 (b) 8 : 7 (c) 5 : 7 (d) 7 : 5	1
13.	If Z is a standard normal variable, then P(0 < Z < 1.7) is equal to : (a) F(0) - F(1.7) (b) F(1.7) - F(0) (c) 1 - F(1.7) (d) F(1.7) - 1	1
14.	A die is rolled thrice. If the event of getting an even number is a success, then the probability of getting at least two successes is : (a) 7/8 (b) ¼ (c) 2/3 (d) 1/2	1
15.	If the variance of a Poisson distribution is 2, then P(X=2) is : (a) $2/e^2$ (b) $2e^2$ (c) $4/e^2$ (d) $4e^2$	1
16.	There are 50 telephone lines in an exchange. The probability that any one of them will be busy is 0.1. The probability that all the lines are busy is : (a) $\frac{5^0 e^{-5}}{\lfloor 0}$ (b) 1 - $\frac{5^0 e^{-5}}{\lfloor 0}$ (c) $\frac{5^{50} e^{-5}}{\lfloor 50}$ (d) 1 - $\frac{5^{50} e^{-5}}{\lfloor 50}$	1
17.	The probability of guessing at least 8 correct answers out of 10 true/false question is : (a) 7/64 (b)7/ 128 (c) 7 /256 (d) 35/1024	1

18.	In Laspeyre's index number weight is Considered as (a) quantity in base year. (b) quantity in current year (c) price in base year (d) price in current year.	1
19.	Given that $\Sigma p_0q_0 = 6600$, $\Sigma p_0q_1 = 8255$, $\Sigma p_1q_0 = 9550$, $\Sigma p_1q_1 = 12010$, where subscripts 0 and 1 are used for base year and current year respectively. The Paashe's index number is : (a) 144.70 (b) 145.49 (C) 145.09 (d) 144.49	1
20.	If $\Sigma p_0 = 141$ $_1$ $\Sigma p_1 = 167$, The price index number by simple aggregative method is (a) 118.44 (b) 0.84 (c) 1·18 (d) 84.4	1

SECTION – B

In this section, attempt any 16 questions out of the Questions 21 - 40.
Each Question is of 1-mark weightage.

21.	In a 1000 m race, A reaches the final point in 56 seconds and B reaches in 70 seconds. By how much distance does A beat B? (a) 100m (b) 120m (c)150m (d) 200m	1
22.	It is currently 8.00 AM. What time (in A.M. or P.M.) will be in next 500 hours? (a) 8 AM (b) 8 PM (c) 4 PM (d) 4 AM	1
23.	A and B invest in a business in the ratio 5 : 6. If 10% of the profit goes to charity and A get Rs.4500 as his share in profit, the total profit is (a)Rs.1000 (b)Rs.12100 (c) Rs.13200 (d) Rs. 9900	1
24.	[(3x7) +5] (mod 4) is (a) 5 (b) 4 (c) 3 (d) 2	1
25.	A, B and C started a business by investing money in the ratio 5 : 7 : 9. After 4 months D joined them by investing money equal to the investment of B. Find the ratio of their profits at the end of a year. (a) 15 : 17 : 28 : 25 (b) 15 : 21 : 27 : 14 (c) 12 : 15 : 17 : 9 (d) 12:17:19:13	1
26.	If A = $\begin{bmatrix} -3 & x \\ y & 5 \end{bmatrix}$ and A = A' , then (a) x = 5, y =-3 (b) x =-3_y = 5 (c) x = y (d) none of these	1

27.	The demand matrix which is consistent with the technology matrix A = $\begin{bmatrix}0.2 & 0.3\\0.4 & 0.1\end{bmatrix}$ and the output matrix X = $\begin{bmatrix}25\\21\end{bmatrix}$ is (a) $\begin{bmatrix}8.9\\3.7\end{bmatrix}$ (b)$\begin{bmatrix}18.7\\13.9\end{bmatrix}$ (c) $\begin{bmatrix}13.7\\8.9\end{bmatrix}$ (d) $\begin{bmatrix}13.9\\8.7\end{bmatrix}$	1
28.	The maximum value of $\frac{\log x}{x}$ is (a) e (b) 2e (c) 1/e (d) 2/e	1
29.	Given that x = sint, y=cost, then value of d^2y/dx^2 is : (a)- tant (b) – sec^2t (c) – sec^3t (d) - sect	1

30.	If the cost function of a certain Commodity is C(x) = 2000 + $50x - \frac{x^2}{5}$, then the average cost of producing 5 units is (a) Rs.451 (b) Rs.450 (c) Rs.449 (d) Rs.2245	1
31.	If the mean and standard deviation of a binomial distribution are 12 and 2 respectively, then the value of its parameter p is (a) 1/4 (b) 1/3 (c) 1/2 (d) 2/3	1
32.	For a binomial variable x, if n=4 and P(x=0) = 16/81 ,then P(x=4) is (A) 1/3 (b)1/27 (c)1/81 (d)1/16	1
33.	A bag contains 2 white and 4 black balls. A ball is drawn 5 times with replacement. The probability that at least 4 of the balls drawn are white is (a) 8/243 (b) 10/243 (c) 11/243 (d) 32/243	1
34.	In a binomial distribution the probability of getting success is 1/4 and the standard deviation is 3. Then its mean is (a) 6 (b) 8 (c) 12 (d) 48	1
35.	Two dice are thrown n times in succession. The probability of obtaining a double six at least once is (A) $\frac{1}{36}^n$ (b) 1- $(\frac{35}{36})^n$ (c) $(\frac{1}{12})^n$ (d) none of these	1

36. 1

Commodity	Rice	Wheat	Fish	Potato	Coal
Price in 2016(p0)	30	22	54	20	15
Price in 2017(p1)	35	25	64	25	18

For this data price index using the price relative method is :

(a)118.44 (b)118.77 (c)118.55 (d) 118.66

37. Given that $\sum p_0q_0$=584, $\sum p_0q_1$=484, $\sum p_1q_0$==623, $\sum p_1q_1$=517, then fisher's price index number is 1

(a)103.29 (b)103.34 (c)107.64 (d)106.74

38. 1

Items	Food	Rent	Cloth	Fuel
Price in 2017	200	100	150	50
Price in 2019	280	200	120	100
Weight	30	20	20	10

The consumer price index for 2019 on the basis of 2017 for data using weighted average of price relative method is :

(a)147.5 (b)146.5 (c)157.6 (d)156.7

39. Index numbers are used for 1

(a) forecasting (b) fixed prices

(c) different prices (d) constant prices

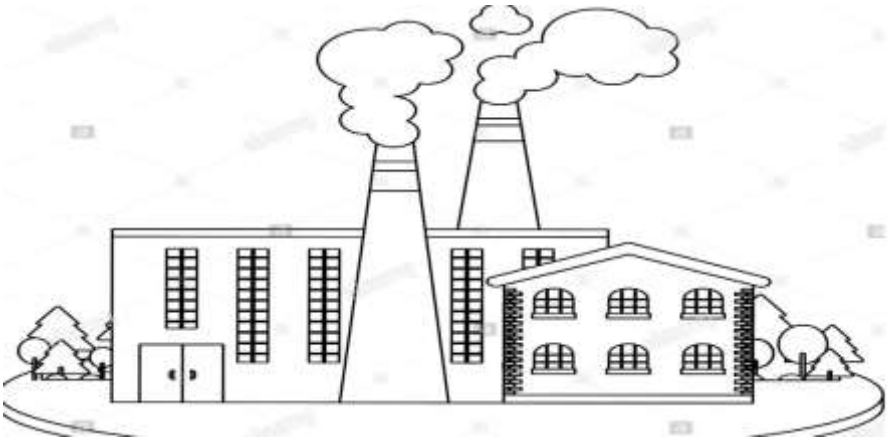
40. For the given 5 values 15, 24, 18, 33, 42 the three years moving averages are 1

(a) 19, 22, 33 (b) 19, 25, 31 (c) 19, 30, 31 (d) 19, 25, 33

SECTION – C

In this section, attempt any 8 questions out 10 Questions.
Each question is of 1 mark weightage.
(Questions 46-50 are based on a Case-Study).

41.	A runs $1\frac{2}{3}$ times as fast as B. If A gives B a start of 80m, how far must the winning post be so that A and B may reach it at the same time. (a)100m (b)120m (c)200m (d)240m	1
42.	Pipes A and B can fill a tank in 4 hour and 5 hour respectively. Another pipe C can empty the full tank in 10 hours. If all the three pipes are opened together then the tank will be filled in (a) $1\frac{2}{7}$ hours (b) $2\frac{6}{7}$ hours (c) $3\frac{5}{7}$ hours (d) $4\frac{6}{7}$ hours	1
43.	If the total cost function is given by C(x)=10x-7x^2+3x^3, then the marginal average cost function (MAC) is given by (a) 10-14x+9 x^2 (b) 10-7x+3 x^2 (c) -7+ 6x (d) -14+18x	1
44.	If x has a Poisson distribution such that P(x=1)=P(x=2) and e^{-2}=0.1353, then P(x=4) is (a) 0.0213 (b)0.9787 (c)0.0902 (d)0.9098	1
45.	The marks obtained in a certain exam follow normal distribution with mean 30 and standard deviation 10. If 1000 students appeared in a exams, The number of students scoring less than 33 marks is (a) 617 (b) 618 (c) 716 (d) 867	1
	Case study	



A industry produces only two goods x and y. The two commodities serve as intermediate input in each others productions. 0.1 unit of x and 0.55 unit of y are needed to produce a unit of x. Where as 0.4 unit of X and 0.2 unit of y are needed to produce a unit of y. For final consumption to 40 units of X and 140 units of y are needed. Based on the above information answer the following questions:

46.	The technology matrix A is (a) $\begin{bmatrix} 0.1 & 0.4 \\ 0.55 & 0.2 \end{bmatrix}$ (b) $\begin{bmatrix} 0.1 & 0.2 \\ 0.55 & 0.4 \end{bmatrix}$ (c) $\begin{bmatrix} 0.2 & 0.4 \\ 0.55 & 0.1 \end{bmatrix}$ (d) $\begin{bmatrix} 0.1 & 0.55 \\ 0.4 & 0.2 \end{bmatrix}$	1
47.	The demand Matrix D is (a) $\begin{bmatrix} 240 \\ 140 \end{bmatrix}$ (b) $\begin{bmatrix} 140 \\ 240 \end{bmatrix}$ (c) $\begin{bmatrix} 100 \\ 140 \end{bmatrix}$ (d) $\begin{bmatrix} 240 \\ 100 \end{bmatrix}$	1
48.	(I-A) is (where I is the identity matrix of the order 2) : (a) $\begin{bmatrix} 0.9 & 0.8 \\ 0.45 & 0.6 \end{bmatrix}$ (b) $\begin{bmatrix} 0.9 & -0.4 \\ -0.55 & 0.8 \end{bmatrix}$ (c) $\begin{bmatrix} 0.8 & -0.4 \\ -0.55 & 0.9 \end{bmatrix}$ (d) $\begin{bmatrix} 0.9 & -0.55 \\ -0.4 & 0.8 \end{bmatrix}$	1
49.	$(I - A)^{-1}$ is (a) $\frac{1}{0.5}\begin{bmatrix} 0.8 & 0.4 \\ 0.55 & 0.9 \end{bmatrix}$ (b) $\frac{1}{0.5}\begin{bmatrix} 0.6 & 0.45 \\ 0.8 & 0.9 \end{bmatrix}$ (c) $\frac{1}{0.5}\begin{bmatrix} 0.9 & 0.55 \\ 0.4 & 0.8 \end{bmatrix}$ (d) $\frac{1}{0.5}\begin{bmatrix} 0.8 & -0.55 \\ -0.4 & 0.9 \end{bmatrix}$	1
50.	The gross output of two commodities are (a) x = 516, y = 496 (b) x = 470, y = 510 (c) x = 496, y = 516 (d) x =510, y = 420	1

Marking Scheme SQP - 4

XII Applied Mathematics

Term - I

Code-241

Q. N.	Correct option	Hints/Solutions
		Section – A
1	a	180 (mod7) =5, (5 is remainder when 180 is divided by 7)
2	c	breadth=x , Length = 2x Perimeter = 2(x+ 2x) ≥ 120 6x≥20 x ≥20 cm.
3	b	Speed of boat in still water = x km/hr speed of stream = y km/hr downstream speed = u= (x+y) km/hr upstream speed = v= (x-y) km/hr then x=(u + v)/2 Here u=16/2=8km/hr , v= 16/4 = 4km/hr x=(8+4)/2 = 6 km/hr
4	a	{4, 11, 18, ……} x=4 (mod 7) x-4 is divided by 7 x-4 = 7k, k is an integer Put k = 0, 1, 2, --- we get positive x = 4, 11, 18,…..
5	d	|-AA'| = |-A||A'| =(-2)(2) = -4
6	d	AB=I |AB|=|I| |B|=$\frac{1}{\|A\|}$=-1/3
7	c	6, -18
8	d	Trivial solution
9	c	50 x=100-4p R=px=$\frac{1}{4}$[100x-x^2] MR = $\frac{d}{dx}$(R) = $\frac{1}{4}$[100-2x) MR=0 ⇒ x=50
10	c	[1/e,∞) $y = x^x$ $\frac{dy}{dx}$ = x^x [logx +1] y is increasing ⇒ $\frac{dy}{dx} \geq 0$ x^x [logx +1] ≥ 0 logx +1 ≥ 0 ⇒ logx ≥ −1 x ≥ e^{-1} x ≥ $\frac{1}{e}$
11	b	12 y=-x^3 +3x^2+ 9x-27

		Slope of curve v=$\frac{dy}{dx}$ For maxima $\frac{dv}{dx}$ = 0 -6x+6=0 x =1 $\frac{d2v}{dx2}$ = -6 x=1 is point of maxima At X=1 slope V is max Max.V at x=1 = -3 $(1)^2$ + 6(1) + 9 =-3+6 +9 = 12
12	b	8 : 7 C.P. of cheaper pulses 'c' = Rs.85 perkg C.P. of dearer pulses 'd' = Rs.100 per kg $\frac{\text{quantity of cheaper pulses}}{\text{quantity of dearer pulses}} = \frac{d-m}{\text{m}-\text{c}}$ =$\frac{100-92}{92-85} = \frac{8}{7}$ where m = mean price of mixture = Rs.92/Kg
13	b	F(1.7) – F(0)
14	d	½ n = 3 , p = ½ , q = ½ (Bernoulli trials) P(X≥2) = P(X=2) + P(X=3)
15	a	$2/e^2$ Variance of a Poisson distribution = λ = 2 P(r)= $\frac{\lambda^r e^{-\lambda}}{\llcorner r}$ = $2/e^2$ Put r = 2
16	c	n=50 , p= 0.1 $\lambda = np$=50 X 0.1=5 P(r)= $\frac{\lambda^r e^{-\lambda}}{\llcorner r}$
17	b	7/128 Binomial distribution B (10, 1/2) n=10 , p = 1/2 P(X ≥ 8) = P(x=8) + P(X=9) + P(X=10)
18	a	Quantity in base year
19	b	145.49 Paasche's index number=$\frac{\Sigma p1q1}{\Sigma p0q1}$ X 100 = $\frac{12010}{8255}$ X 100 =145.49
20	a	118.44 By simple aggregate method P_{01} =$\frac{\Sigma p1}{\Sigma p0}$ X 100 = $\frac{167}{141}$ X 100 = 118.44

		Section –B

21	d	200m Speed of B = $\frac{1000}{70}$ m/sec. Distance Covered by B after A has finished the race Distance Covered by B in (70-56) = 14 sec. = $\frac{100}{7}$ X 14 = 200m 200m
22	d	4:00 A.M. 500 (mod 24) ≡ 20 (mod 24) (500=24X20 + 20) 500 hours is equivalent to 20 hours Now 8+ 20 = 4 (mod 24) Hence it will be 4:00 A.M.
23	a	Rs11,000 A:B = 5:6 Total 5+6=11 Let total profit = Rs. x $\frac{5}{11}$ of (90% of x) = 4500 x = Rs11,000 (Charity = 10%.)
24	d	2 [(3x7) +5] (mod4) ≡ 26 (mod 4) ≡ 2 (mod 4)
25	b	15:21:27:14 Let invested amount of A, B and C are 5x , 7x and 9x. D joined with 7x (equal to B's invest). A, B &C invested for 12 months whereas D invested for 12-4 = 8 months ∴ A : B : C : D = 5x X 12 :7x X 12 : 9x X 12 : 7x X 8 =15:21:27:14
26	(c)	X=y A =A' $\begin{pmatrix} -3 & x \\ y & 5 \end{pmatrix} = \begin{pmatrix} -3 & x \\ y & 5 \end{pmatrix}$ x=y
27	(c)	$\begin{bmatrix} 13.7 \\ 8.9 \end{bmatrix}$ AX+D= X D=(I-A)X $= \begin{pmatrix} 0.8 & -0.3 \\ -0.4 & 0.9 \end{pmatrix} \begin{bmatrix} 25 \\ 21 \end{bmatrix} = \begin{pmatrix} 13.7 \\ 8.9 \end{pmatrix}$
28	(c)	1/e Y=(logx)/x dy/dx=[x*1/x- (logx).1]/x^2 for maximum y, dy/dx=0 (1-logx) / x^2=0 logx=1 x=e max y = (log e) /e = 1/ e

29	(c)	-sec^3t X=sin t dx/dt= cos t Y=cos t dy/dt= -sin t dy/dx= =- tan t d2y/dx2 = - sec^2t (1/cos t) = -sec^3t
30	(c)	Rs 449 AC = C(x)/x=C(5)/5 (here x = 5) = $\frac{2000+250-1/5*25}{5}$
31	d	2/3 Binomial distribution mean =np =12 Standard deviation = √npq=2 implies npq=4 12q=4 q=1/3 p= 1-q =2/3
32	c	1/81 P(X=0) = $^nc_0$ P^0 q^4 =$^4c_0$ q^4 (n=4) P(X=0) =16/81 = q^4 q= 2/3 p=1-2/3 , p=1/3 P(X = 4) = 1/81
33	c	11/243 p= 2/(2+4), p=1/3, q =2/3 , n=5 P(X≥4) = P(X=4) + P(X=5) = $1/3^5$ (2 $^5c_1$ + $^5c_1$) =11/243
34	c	12 Binomial distribution p=1/4, q=3/4 Standard deviation =√n p q=3 implies n p q=9 Mean= np= 9/q = (9/3)*4 np=12
35	b	1-(35/ 36)^n X= Number of double six Success= getting double six P=P(s)=1/36 q=35/36 P(X≥1) =1-P(X=0) =1- $^nc_0$ p^0q^{36} =1-(35/36)^n
36	b	

Commodity	Rice	Wheat	Fish	Potato	Coal
Price in 2016(p0)	30	22	54	20	15
Price in	35	25	64	25	18

		2017(p1)					
		p1/p0	116.67	113.64	118.52	125	120

118.77

$P_{01}=1/N\ (\sum(P_1/P_0)\ X\ 100) = 593.83/5 = 118.77$

37	d	106.74 Fisher's price index number $P_{01}= (\sqrt{\sum p_1q_0 / \sum p_0q_0}\ x\ \sum p_1q_1 / \sum p_0q_1)\ X100$ $= (\sqrt{623/584}\ x\ 517/484)\ X100 = 106.74$
38	A	147.5

Items	Food	Rent	cloth	fuel
$I = P_1/P_0\ X100$	280/200 X100 =140	200/100 X100 =200	150/120 X 100 =80	100/50 X100 =200
I W	4200	4000	1600	2000

$\sum IW=11800, \sum W=80$ Index Number= $\sum IW/\sum W=11800/80 = 147.5$

39	a	Forecasting
40	b	19,25,31 15+24+18 =57 then 1/3 of it is 19, 24+18+33=75 then 1/3 of75 is 25, 18+33+42=93 then 1/3 is of 93 is 31.
		Section C
41	c	200 m In same time A covers x metres (say d_1) and B covers (x-80) metres,(d_2) A & B takes equal time Let distance of winning pole = x metres, speed of B (s_1) = y m/sec , speed of A (s_2)= 5/3 of y x / (5/3) y= (x-80) y 3x=5(x-80) x=200 m

42	b	20/7 hours Pipe A can fill in 1 hour = 1/4 part of tank Pipe B can fill in 1 hour = 1/5 part of tank Pipe B can empty in 1 hour = 1/10 part of tank (A+B-C) in 1 hour=(1/4 +1/5-1/10) part=7/20 part total time to fill tank= 20/7 hours
43	c	-7+6x $C(x)=10x-7x^2+3x^3$ $AC = C(x) / x =10-7x+3x^2$ MAC =d(AC)/dx=-7+6x
44	c	0.0902 $P(X=r) = e^{-\lambda}. \lambda^r/r!$ $P(X=1) = e^{-\lambda}. \lambda^1/1!$ $P(X=2) = e^{-\lambda}. \lambda^2/2!$ $P(X=1) = P(X=2)$ $e^{-\lambda}. \lambda^1/1= e^{-\lambda}. \lambda^2/2!$, then solving for λ, $\lambda=2$ $P(X=4) = e^{-2}.2^4 /4!$ $= 0.1353x16 / 24 = 0.0902$
45	b	618 $\mu=30$, sigma =10, $Z=(X-30)/10$, where X = marks obtained P (X less than 33)= P(Z less than (33-30)/10) =P (Zlessthan0.3) =F (0.3) =0.6179 using table Number of students (X less than 33) =1000 x .6179 =617.9 i.e 618
46	a	$\begin{pmatrix} 0.1 & 0.4 \\ 0.55 & 0.2 \end{pmatrix}$ $A=\begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix}$ $a_{ij}=x_{ij}/x_i$
47	a	$\begin{bmatrix} 240 \\ 140 \end{bmatrix}$
48	c	$\begin{pmatrix} 0.9 & -0.4 \\ -0.55 & 0.8 \end{pmatrix}$
49	a	$1/0.5 \begin{pmatrix} 0.8 & 0.4 \\ 0.55 & 0.9 \end{pmatrix}$ $(I-A)^{-1}= (1/ \lvert I-A \rvert) (adj (I-A)$
50	c	X=496, Y=516 $X = (I-A)^{-1}D$ $X= 1/0.5 \begin{pmatrix} 0.8 & 0.4 \\ 0.55 & 0.9 \end{pmatrix} \begin{pmatrix} 240 \\ 140 \end{pmatrix} = \begin{pmatrix} 496 \\ 516 \end{pmatrix}$

Sample Question Paper-5

CLASS: XII

Session: 2021-22

Applied Mathematics (Code-241)

Term - 1

Time Allowed: 90 minutes **Maximum Marks: 40**

General Instructions:

1. This question paper contains three sections – A, B and C. Each part is compulsory.
2. Section - A has 20 MCQs, attempt any 16 out of 20.
3. Section - B has 20 MCQs, attempt any 16 out of 20
4. Section - C has 10 MCQs, attempt any 8 out of 10.
5. There is no internal choice in any section.
6. All Questions carry equal Marks.

	SECTION – A In this section, attempt any 16 questions out of Questions 1 – 20. Each Question is of 1 mark weightage	
1	Find 5^6 (mod 4). (a)1 (b) 2 (c) 3 (d) 4	1
2	What time will it be after 1250 hours, If the present time is 9:00 pm? (a)10:00 pm (b) 11 pm (c) 10 :00 am (d) 11:00 am	1
3	A container contains 40 litre milk. From this container 4 litre milk was taken out and replaced with water. This process was repeated further two more times. How much milk is there in the container now?(Approx) a) 9Lt b) 19Lt c) 29LT d) 39Lt	1
4	A man rows 15 km upstream and 25 km downstream in 5 hours each time. What is the speed of the current (in km/hr)? a) 4 b) 3 c) 2 d) 1	1
5	-31 mod 7 ,is equal to a) 4 b) -3 c) 0 d) 3	1
6	For Matrices A(3 × 3) and B(3 × 3), the value of (AB)' = a) B'+A' b) B'A' c) A' B' d) A'+ B'	1
7	If A is a square matrix of order 3 and $\|A\| = -3$, then $\|adj(A)\|$ is equal to a) 4 b) 3 c) 9 d) 1	1

8	Find k, if A=$\begin{bmatrix} -2 & 3 \\ k & 4 \end{bmatrix}$ is a singular matrix a) 5/3 b) 8/ 3 c) -5/3 d) -8/3	1
9	A toy manufacturing firm assesses its variable cost to be 'x' times the sum of 30 and 'x', where 'x' is the number of toys produced, also the cost incurred on storage is Rs1500 Find the total cost function and the marginal cost when 20 toys are produced. a) 70 b) 30 c) 90 d) 10	1
10	Slope of the normal at a point (2,6) to the curve $y = x^3 - x$ a) -2/11 b) -1/11 c) 1/11 d) 2/11	1
11	What is minimum value of function f(x)= $9x^2 + 12x + 2$ a) -4 b) - 3 c) -2 d) -1	1
12	In a binomial distribution having 'n' number of Bernoulli trials where p denotes the probability of success and q denotes the probability of failure then Variance = a) n/pq b) np/q c) n+p+q d) npq	1
13	If a fair coin is tossed 9 times, find the probability of exactly five tails a) 63/256 b) 64/256 c) 16/256 d) 36/256	1
14	In a poisson distribution, if mean is 2, what is the variance? (a) 1 b) 2 c) 3 d) 4	1
15	Given that mean of a normal variate X is 12 and standard deviation is 4, then find the Z-Score of data point 20 a) 4 b) 3 c) 2 d) 1	1
16	Given that the scores of a set of candidates on an IQ test are normally distributed. If the IQ test has a mean of 100 and a standard deviation of 10, what is the probability that a candidate who takes the test will score between 90 and 110 a) 0.9826 b) 0.8826 c) 0.7826 d) 0.6826	1
17	What is the index number of the base period? a) 100 b) 10 c) 1 d) 1000	1

18	In Paasche's price index number weight is considered as a) Quantity in base year b) Quantity in current year c) Prices in base year d) Prices in current year	1
19	Which index number is called as ideal index number a) Laspeyres b) Paasches c) Fishers d) None of the above	1
20	Cost of living at two different cities can be compared with the help of: a) Value index b) Un-weighted index c) Volume index d) Consumer price index	1

SECTION – B

In this section, attempt any 16 questions out of the Questions 21-40. Each question is of 1-mark weightage.

21	*In a 1000 m race, A reaches the final point in 56 seconds and B reaches in 70 seconds. By how much distance does A beat B?* *(a) 100 m (b) 120 m (c) 150 m (d) 200 m*	1
22	*It is currently 12:00 Noon, What time (in A.M. or P.M.) will be in next 500 hours?* *(a) 8:00 AM (b) 8:00PM (c)4:00 AM (d) 4:00PM*	1
23	A, B and C enter into partnership in the ratio $\frac{7}{2} : \frac{4}{3} : \frac{6}{5}$. After 4 months A increases his share by50%. If the total profit at the end of a year is ` 21600, then find B's share in the profit. (a) Rs 4000 (b) Rs 5000 (c) Rs 4500 (d)Rs 5500	1
24	The cost price of type I rice is ` 60 per kg and that of type II is ` 80 per kg. If both types of rice are mixed in the ratio 2 : 3 respectively, then find the price per kg of the mixed rice. *(a)* Rs 75 (b) Rs 72 (c) Rs. 73 (d) Rs 74	1
25	Pipe A can fill the tank 2 times faster than pipe B. If both pipes A and B running together can fill the tank in 24 minutes, find how much time will pipe B alone take to fill the tank? (a) 70 Min (b) 73 min (c) 71 Min (d) 72 min	1

26	*If* $A = \begin{bmatrix} 1 & 2a \\ -8 & b+1 \end{bmatrix}$ *and* $B = \begin{bmatrix} 1 & -6 \\ -8 & 13 \end{bmatrix}$ *. If A and B are equal matrices, find the values of a and b.* (a) $a = 3,\ b = 12$ (b) $a = 3,\ b = -12$ (c) $a = -3,\ b = 12$ (d) $a = -3,\ b = -12$	1
27	In which of the technology matrix, Hawkins- Simon conditions are satisfied (a) $\begin{bmatrix} 0.3 & 0.4 \\ 0.7 & 1.4 \end{bmatrix}$ (b) $\begin{bmatrix} 0.1 & 0.5 \\ 0.8 & 1.2 \end{bmatrix}$ (c) $\begin{bmatrix} 1.03 & 0.6 \\ 0.3 & 1.5 \end{bmatrix}$ (d) $\begin{bmatrix} 0.5 & 0.4 \\ 0.1 & 0.6 \end{bmatrix}$	1
28	The function $f(x) = \lvert x \rvert$ is (a) differentiable at $x = 0$ (b) differentiable everywhere (c) differentiable everywhere except at $x = 0$ (d) none of the above	1
29	If $x = t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ is equal to (a) $\frac{3}{2}$ (b) $\frac{3}{4t}$ (c) $\frac{3}{2t}$ (d) $\frac{3}{4}$	1
30	The price per unit of a commodity produced by a company is given by $p = 30 - 2x$ and 'x' is the quantity demanded. Find the marginal revenue, when 5 commodities are in demand (or produced). (a) Rs 10 (b) Rs 12 (c) Rs 50 (d) Rs 26	1
31	*What is the mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face?* *(a) 6 (b) 12 (c) 2 (d) 4*	1
32	*In a manufacturing unit inspection, from a lot of 20 baskets which include 6 defectives, a sample of 2 baskets is drawn at random with replacement, then Var(X) for the random variable X (where X=Number of defective baskets)is* *(a) 0.32 (b) 0 .42 (c) 0.84 (d) 0.12*	1
33	*The monthly salaries of workers in a certain factory are normally distributed. The mean salary is Rs 4000 and standard deviation is Rs 450. If 668 workers are getting salary less than Rs 3325, find the total number of workers in the factory.* *(a) 12000 (b) 11000 (c) 10000 (d) 13000*	1
34	*In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear a hurdle is* $\frac{5}{6}$ *. What is the probability that he will knock down fewer than 2 hurdles?* *(a)* $\frac{5^{10}}{2\times6^9}$ *(b)* $\frac{6^{10}}{2\times5^9}$ *(c)* $\frac{5^{10}}{2\times6^9}$ *(d)* $\frac{2^{10}}{5\times6^9}$	1

35	*At a telephone enquiry system, the number of phone calls regarding relevant enquiry follow Poisson distribution with an average of 5 phone calls during 10-minute time intervals. The probability that there is at most one phone call during a 10-minute time period is* *(a)* $\frac{6}{55}$ *(b)* $\frac{6}{5}$ *(c)* $\frac{6}{e^5}$ *(d)* $\frac{6}{5^e}$	1
36	*If Laspeyre index number =160, Passche's index number = 90, then Fisher's index number is* *(a) 90 (b) 120 (c) 140 (d) 160*	1
37	*The price and quantity of group of commodities is given in the following table:* Commodity / Price / Quantity 2008 (P_0) / 2012 (P_1) / 2008 (Q_0) / 2012 (Q_1) Rice 10 13 4 6 Wheat 15 18 7 8 Rent 25 29 5 9 Fuel 11 14 8 10 Fisher's price index number for the given data is (a) 120.1 (b) 121.1 (c) 121.2 (d) 122.1	1
38	*Following table shows the data on energy consumption and expenditure at Badarpur Thermal Power Station, in Delhi region. Find an aggregative price index for the energy expenditures in year 2015 using Laspeyres' index number.* ***Sector*** / ***Quantity (weights)*** / ***Unit Price (Rs /kwh)*** *Year 1987* / *Year 2015* / *Year 1987* / *Year 2015* *Commercial* 5416 6015 1.97 10.92 *Residential* 15293 20262 2.32 6.16 *Industrial* 21287 17832 0.79 5.13 *Agriculture* 9473 8804 2.25 8.10 *(a) 403 (b) 401 (c) 402 (d) 400*	1
39	Which of the following index number satisfy the time reversal test ? a) Laspeyre's b) Paasche's c) Fisher's d) None of the above	1
40	In Laspeyre's price index number weight is considered as (a) Quantity in base year (b) Quantity during current year (c) Prices in base year (d) Prices in current year.	1

	SECTION – C **In this section, attempt any 8 questions out of 10 Questions.** **Each question is of 1 mark weightage.** **(Questions 46-50 are based on a Case-Study).**	
41	A retailer has 250 kg of rice, a part of which he sells at 10% profit. The remaining quantity of rice is of low quality and he sold it at 5% loss. Overall he made a profit of 7%. Find the quantity of rice sold at 5% loss a) 50 kg b) 40kg c) 70kg d) 10 kg	1
42	A person can row a boat at a speed of 5 km / hour in still water. It takes him thrice as long to row upstream as to row downstream. Find the rate at which the stream is flowing in km/hr a) 4 b) 2.5 c) 2 d) 1.5	1
43	In a 1000 metres race, A defeats B by 100 metres and B defeats C by 100 metres. In same race by how many metres will A defeat C ? a)100 b) 150 c) 200 d) 190	1
44	A traffic engineer records the number of bicycle riders that use a particular cycle track. He records that an average of 3.2 bicycle riders use the cycle track every hour. Given that the number of bicycles that use the cycle track follow a Poisson distribution, what is the probability that: 2 or less bicycle riders will use the cycle track within an hour? a) 0.011 b) 0.021 c) 0.031 d) 0.381	1
45	For Binomial distribution B (4, 2/3) , the value of variance is a) 0.89 b) 090 c) 0.91 d) 0.99	1
	Case study The economy of a state is composed of various sectors. To understand the basic concept, we consider two sectors coal mining (sector 1) and utilities (sector 2). The coal mining produces coal and utilities produces electricity. Assume that these products are measured by their rupee value. By one unit of product we mean 1 rupee worth of that product. To produce Re1 worth of coal the coal mining sector uses Rs0.50 of coal and Rs0.10 of electricity. To produce Re1 worth of electricity the utilities sector uses Rs0.25 of coal and Rs0.25 of electricity.	

	  **Based on the above information, answer the following questions:**	
46	The technology coefficient matrix A is *(a)* $\begin{bmatrix}0.50 & 0.10\\0.25 & 0.25\end{bmatrix}$ *(b)* $\begin{bmatrix}0.50 & 0.25\\0.10 & 0.25\end{bmatrix}$ *(c)* $\begin{bmatrix}0.25 & 0.25\\0.50 & 0.10\end{bmatrix}$ *(d)* $\begin{bmatrix}0.10 & 0.50\\0.25 & 0.25\end{bmatrix}$	1
47	*The matrix* $(I-A)^{-1}$*is* *(a)* $\frac{1}{8}\begin{bmatrix}15 & 5\\2 & 10\end{bmatrix}$ *(b)* $\frac{1}{7}\begin{bmatrix}15 & 2\\5 & 10\end{bmatrix}$ *(c)* $\frac{1}{7}\begin{bmatrix}15 & 5\\2 & 10\end{bmatrix}$ *(d)* $\frac{20}{7}\begin{bmatrix}0.75 & 0.25\\0.50 & 0.10\end{bmatrix}$	1
48	The system is viable because (a) $\lvert I-A\rvert>0$ and diagonal elements of $(I-A)<0$ (b) $\lvert I-A\rvert>0$ and diagonal elements of $(I-A)>0$ (c) $\lvert I-A\rvert<0$ and diagonal elements of $(I-A)>0$ (d) $\lvert I-A\rvert<0$ and diagonal elements of $(I-A)<0$	1
49	If there is external demand worth ₹7000 of coal and ₹14000 of electricity, then production of two sectors to meet the demand is (a) Rs 25000 of coal, Rs 22000 of electricity (b) Rs 12000 of coal, Rs 20000 of electricity (c) Rs 15000 of coal, Rs 22000 of electricity (d) Rs 27000 of coal, Rs 22000 of electricity	1
50	How much worth of coal and electricity is used internally? (a) Rs 25000 of coal, Rs 22000 of electricity (b) Rs 22000 of coal, Rs 15000 of electricity (c) Rs 20000 of coal, Rs 10000 of electricity (d) Rs 18000 of coal, Rs 8000 of electricity	1

Sample paper-5
Marking scheme
Applied mathematics
Term-I
Code-241

Q.N	Correct option	Hints/ solutions

Section-A		
1	a	$5 \equiv 1 \pmod 4 = 1$ $5^6 \equiv 1^6 \pmod 4$ (Using property 5) Hence, $5^6 \pmod 4 = 1$
2	b	Number of hours in a day = 24 hours $1250 \pmod{24} = 2$ (as 1250 hours = 52 days + 2 hours) Therefore, it will be 11:00 pm after 1250 hours
3	c	Total milk in container = 40 l Milk taken out = 4 l No. of times process repeated = 3 Milk contained by the container now $=X\left(1-\frac{Y}{X}\right)^n$ unit where x is total quantity, y is quantity removed, n is no. of times operation repeated. = I.e. 4 $40\left(1-\frac{3}{40}\right)^3 = 29$ lit approx.
4	d	Let the speed of the boat in still water be x km/hr and the speed of the current be y km/hr then (x+y) × 5 = 25 and (x-y) × 5 = 15 (Distance = Speed x Time) x + y = 5 and x – y = 3 On solving we get, x = 4 and y = 1 Thus, speed of the current is 1 km/hr
5	a	$-31=7\times(-5)+4$
6	b	
7	c	$\lvert adj(A)\rvert = \lvert A\rvert^{n-1}$ (n=order of matrix) $(-3)^2=9$
8	d	$A=\begin{bmatrix}-2 & 3\\ k & 4\end{bmatrix}$ For singular matrix $\lvert A\rvert=0$ $-8-3k=0$ $K=-8/3$
9	a	AThe total cost function C(x) is given by, C(x) = x (x + 30) + 1500 = x^2 + 30x + 1500 The marginal cost MC is given by c'= 2x+ 30 Marginal cost of producing 20 toys is 2(20)+30 The marginal cost of producing 20 toys is Rs 70

10	b	y' =$3x^2$-1 Slope= $3(2)^2$-1=11 Slop of normal=-1/11
11	c	$9x^2 + 12x + 2$ $= (3x + 2)^2$**-2** so minimum value is -2
12	d	Variance =npq
13	a	Repeated tosses of a fair coin qualify as Bernoulli's trails Let X denote the number of tails in an experiment of 9 such trials and hence is the binomial distribution Here, n = 9, p = $\frac{1}{2}$ and q = 1 – p = $\frac{1}{2}$ As P ('r' successes) = $C_r^n p^r q^{n-r} = \frac{n!}{r!(n-r)!} p^r q^{n-r}$ Probability of exact 5 successes in 9 trials = P(X = 5) = $C_5^9 p^5 q^{9-5}$ $= \frac{9!}{5!(9-5)!} \cdot \left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{2}\right)^{9-5}$ $= \frac{9!}{5!(9-5)!} \cdot \left(\frac{1}{2}\right)^9$ $= \frac{63}{256}$
14	b	
15	c	Z= $\frac{x-\mu}{\sigma}$ Z= $\frac{20-12}{4}$ =2
16	d	P(90 < X < 110) = P(X < 110) - P(X < 90) P(90 < X < 110) = P (–1 < Z < 1) = P (Z < 1) – P(Z < –1) = 0.8413 – 0.1587 = 0.6826
17	a	100
18	b	Quantity in current year
19	c	Fisher
20	d	Consumer price index

Section-B

21	D	Since A is faster by 14 secs. Therefore, he beats B by= $\frac{1000}{70} \times 14 = 200\ m$
22	a	we find 500 (mod 24)

		500 = 24 × 20 + 20, so 500 ≡ 20 (mod 24) 500 hours is equivalent to 20 hours. Hence, it will be 8:00 A.M. in next 500 hours
23	A	Given, ratio of A, B and C's investment = $\frac{7}{2} : \frac{4}{3} : \frac{6}{3}$ = 105 : 40 : 36 Let the initial investment of A, B and C be ` $105x$, ` $40x$ and ` $36x$ respectively. A increases his share by 50% after 4 months, ∴ A's share of profit : B's share profit : C's share of profit $= (105x \times 4 + 105x \times 8) : 40x \times 12 : 36x \times 12$ $= 1680x : 480x : 432x$ =35 : 10 : 9 Hence, B's share of profit = $\frac{10}{54} \times 21600 = 4000$
24	B	$\frac{2}{3} = \frac{80-m}{m-60}$ $\Rightarrow 2m - 120 = 240 - 3m \Rightarrow 5m = 360$ $\Rightarrow m = 72$
25	D	$\frac{1}{x} + \frac{2}{x} = \frac{1}{24} \Rightarrow x = 72$
26	C	2a = -6 $\Rightarrow$ a = -3 and b + 1 = 13 $\Rightarrow$ b = 12
27	D	$\lvert I - A \rvert$ =0.16 > 0 and main diagonal of $(I - A)$ is positive.
28	C	
29	B	$\frac{dy}{dx} = \frac{3t}{2}, \frac{d^2y}{dx^2} = \frac{3}{2}\frac{dt}{dx} = \frac{3}{4t}$
30	A	$R = px = (30 - 2x)x = 30x - 2x^2$ The marginal revenue $\frac{dR}{dx} = 30 - 4x$ The marginal revenue of producing 5 commodities is $30 - 4 \times 5 = 10$
31	C	$P(1) = \frac{3}{6} = \frac{1}{2}, P(2) = \frac{2}{6} = \frac{1}{3}, P(5) = \frac{1}{6}$ $\Sigma p_i x_i = \frac{1}{2} \times 1 + 3 \times 2 + 6 \times 5 = 6 = 2$

32	B	*X = 0, 1 and 2 , n = 2 If drawing a defective basket is considered a success, then p =* $\frac{6}{20} = 3$ *,* $q = 1 - \frac{3}{10} = \frac{7}{10}$*, Var(X) =* $npq = 2 \times \frac{3}{10} \times \frac{7}{10} = 0.42$
33	C	$Z = \frac{X - 4000}{450}$, $P(X < 3325) = P\left(Z < \frac{3325 - 4000}{450}\right)$ $= P(Z < -1.5)$ $= F(-1.5) = 1 - F(1.5) = 1 - 0.9332 = 0.0668$ According to given $0.0668 \times n = 668 \Rightarrow n = 10000$
34	A	p = $\frac{1}{6}$, q = $\frac{5}{6}$, n = 10 Required probability = P(knocking down less than 2 hurdles) = P(0) + P(1) $= C_0^n q^{10} + C_1^n pq^9 = (q + 10p)q^9 = \left(\frac{5}{6} + 10 \times \frac{1}{6}\right) \times \left(\frac{5}{6}\right)^9$ $= \frac{5^{10}}{2 \times 6^9}$
35	C	m=5, P $(x = 0,1) = e^{-5}\frac{5^0}{\lfloor 0} + e^{-5}\frac{5^1}{\lfloor 1} = e^{-5}(1 + 5) = \frac{6}{e^5}$
36	B	Fisher's index number = $\sqrt{L.I \times P.I} = \sqrt{160 \times 90} = 120$
37	B	$\Sigma P_0 Q_0 = 359$, $\Sigma P_0 Q_1 = 515$, $\Sigma P_1 Q_0 = 435$, $\Sigma P_1 Q_1 = 623$ Fishers index number = $\sqrt{\frac{435}{359} \times \frac{623}{515} \times 100} = 121.2$
38	A	*Laspeyres' index number* $I_{2015} = \frac{10.92 \times 5416 + 6.16 \times 15293 + 5.13 \times 21287 + 8.10 \times 9473}{1.97 \times 5416 + 2.32 \times 15293 + 0.97 \times 21287 + 2.25 \times 9473} \times 100$ =403
39	C	
40	a	

41	a	
42	b	Let the rate at which the stream is flowing be x km/hr and let the distance covered by the boat be y km. According to the question $\frac{3y}{5+x} = \frac{y}{5-x}$ $\Rightarrow$ 3 (5 – x) = 5+x $\Rightarrow$ 15 – 3x = 5+x $\Rightarrow$ 4x = 10 $\therefore$ x = 2.5 The stream is flowing at the rate of 2.5 km/h
43	d	A:B:C=1000:900:810

44	d	For this problem, E(X) = Var(X) = λ = 3.2 The goal is to find P(X ≤2) As $P\ (X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$ $\Rightarrow P(X = 0) = \frac{3.2^0 e^{-3.2}}{0!} = \frac{1}{e^{3.2}} = 0.041$ $P(X = 1) = \frac{3.2^1 e^{-3.2}}{1!} = \frac{3.2}{e^{3.2}} = 0.13$ $P(X = 2) = \frac{3.2^2 e^{-3.2}}{2!} = \frac{5.12}{e^{3.2}} = 0.21$ Therefore, P(X ≤ 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.041+0.13+0.21 = 0.381
45	a	Variance =npq $4x\frac{2}{3}x\frac{1}{3}=8/9=0.89$
46	b	Science elements of technology matrix a_{ij}, represents units of sector i to produce 1 unit of sector j
47	c	
48	b	system is viable if $\|I - a\| > 0\ and\ 1 - a_{11} > 0, 1 - a_{22} > 0$
49	a	$X=(I - A)^{-1}D$
50	d	Internal consumption = total production – external demand=X-D

Question Bank (Chapter-1)

Numbers, Quantification and Numerical Applications

MCQ's (1 to 20):

1. $(49 + 57)(\text{mod } 50)$ is

   $(a)\ 4$ $(b)\ 5$ $(c)\ 6$ $(d)\ 7$.

2. $(15 - 53)(\text{mod } 4)$ is

   $(a)\ 1$ $(b)\ 2$ $(c)\ 3$ $(d)\ 4$.

3. $(18 \times 10)(\text{mod } 7)$ is

   $(a)\ 5$ $(b)\ 4$ $(c)\ 3$ $(d)\ 2$.

4. $[(3 \times 7) + 5](\text{mod } 4)$ is

   $(a)\ 5$ $(b)\ 4$ $(c)\ 3$ $(d)\ 2$.

5. $(-6 \times 5)(\text{mod } 7)$ is

   $(a)\ -2$ $(b)\ 5$ $(c)\ -5$ $(d)\ 2$.

6. (8×14) in 12 hours clock is

   (a) 4 O' clock (b) 8 O' clock (c) 6 O' clock (d) 2 O' clock .

7 . $(09:30 + 16:40)$ in 24 hours clock is

(a) $03:10$ (b) $26:10$ (c) $2:10$ (d) $25:70$.

8. $x \equiv 4(\text{mod}7)$, then positive values of are

(a) $\{4, 11, 18, \ldots\}$ (b) $\{11, 18, 25, \ldots\}$ (c) $\{4, 8, 12, \ldots\}$ (d) $\{1, 8, 15, \ldots\}$.

9. $\sigma(15)$ is equal to

(a) 8 (b) 24 (c) 4 (d) 16 .

10. $\omega(32)$ is equal to

(a) 1 (b) 3 (c) 5 (d) 7 .

11. $\Omega(32)$ is equal to

(a) 1 (b) 3 (c) 5 (d) 7 .

12. Three types of wheat costing Rs 18 per kg, Rs 20 per kg and Rs 25 per kg are mixed together. If mixed variety is sold at Rs 22 per kg, the in which these of wheat should be mixed respectively is

(a) $1:2:3$ (b) $2:2:3$ (c) $2:3:1$ (d) $1:1:2$.

13. In what ratio must a grocer mix two varieties pulses costing Rs 85 per kg and Rs 100 per kg respectively so as to get a mixture worth Rs 92 per kg?

(a) $7:8$ (b) $8:7$ (c) $5:7$ (d) $7:5$.

14. In what ratio must water be mixed with milk to gain $16\frac{2}{3}\%$ on selling the mixture at cost price?

(a) $1:6$ (b) $6:1$ (c) $3:2$ (d) $2:3$.

15. Milk and water in two vessels A and B are in the ratio $5:3$ and $5:4$ respectively. In what ratio the liquid of both the vessels be mixed to obtain a new mixture in which milk and water is $7:5$ respectively?

(a) $3:2$ (b) $3:5$ (c) $2:3$ (d) $2:5$.

16. A jar full of whisky contains 40% alcohol. A part of this whisky is replaced by another containing 19% alcohol and now percentage of alcohol is found to be 26%. The quantity of whisky replaced is

(a) $\frac{1}{3}$ part (b) $\frac{2}{3}$ part (c) $\frac{2}{5}$ part (d) $\frac{3}{5}$ part .

17. A merchant has 1000 kg of sugar. Part of which he sells at 8% profit and rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit is

(a) 400 kg (b) 600 kg (c) 560 kg (d) 640 kg .

18. A boy's speed with current is 15 km/h and the speed of the current is 2.5 km/h. The boy's speed against the cuurent is

(a) 8.5 km/h (b) 9 km/h (c) 10 km/h (d) 12.5 km/h .

19. A boat running downstream covers 16 km in 2 hours while for covering the same distance upstream it takes 4 hours. What is the speed of the boat in still water?

(a) 4 km/h (b) 6 km/h (c) 8 km/h (d) 10 km/h .

20. The speed of a boat in still water is 15 km/h.and the rate of current is 3 km/h. The distance travelled by boat in 12 minutes is

(a) 1.2 km (b) 1.8 km (c) 2.4 km (d) 3.6 km .

Assertion – Reasoning Questions

DIRECTION: In the following questions, a statement of assertion (A) is followed by a statement of reason (R), mark the correct choice as:

(a) Both assertion and reason are the true and reason is the correct explanation of assertion.
(b) Both assertion and reason are true but reason is not the correct explanation of assertion.
(c) Assertion is true but reason is false.
(d) Assertion is false but reason is true.

21. **Assertion:** The least positive value of x such that $71 \equiv x(mod 8)$ is 7.

    Reason: if $a \equiv b(mod m), then\ m|(a-b)$.

22. **Assertion:** Today is Tuesday. My uncle will come after 43 days. My uncle will come on Monday.
    Reason: since there are 7 days in a week so, we use the method $43 \equiv x(mod 7)$.
23. **Assertion:** The value of $\tau(80)$ is 10.
    Reason: $\omega(n)$ = number of prime divisors of n.
24. **Assertion:** The highest power of 3 in $\lfloor 1000$ is 498.
    Reason: $\sigma(n)$ =sum of divisors of n.
25. **Assertion:** the value of $\sigma_2(6) = 50$
    Reason: The value of $\sigma(6) = 12$
26. **Assertion:** The value of $\omega(12)$ is 2.
    Reason: $\omega(n)$ = number of prime divisors of n.
27. **Assertion:** $\emptyset(5) = 4\ and\ \emptyset(6) = 2$.
    Reason: $\emptyset(n)$ =Number of relatively primes to n and not exceeding to n.
28. **Assertion:** The highest power of 6 in $\lfloor 245$ is 121.
    Reason: The highest power of 6 in $\lfloor 245$ =min $(power\ of\ 2, power\ of\ 3)$.

Case Study based Questions:

29. Today in class mathematics, Mr. Gopal is going to teach about the arithmetic function. He teaches different functions such as $\tau(n), \sigma(n), \omega(n), and\ \varphi(n)$ to the class. Two students Reena and Meeta told 'sir, please give us a number and we calculate the values of different arithmetic functions for it. Mr. Gopal give the number 9 to Meeta and 12 to Reena.

Now according to the given information answer the following questions:

(i) $\tau(n)$ denotes to:
(a) sum of positive divisors of n.
(b) number of positive divisors of n
(c) number of coprime numbers of n
(d) none of these

(ii) The value of $\tau(12)$ is:
(a) 6 (b) 3 (c) 4 (d) 5

(iii) $\sigma(n)$ denotes:
(a) sum of positive divisors of n.
(b) number of positive divisors of n
(c) number of coprime numbers of n
(d) none of these

(iv) the value of $\sigma(9)$ is:
(a) 10 (b) 11 (c) 12 (d) 13

(v) The value of $\omega(9)$ is:
(a) 1 (b) 2 (c) 3 (d) none of these

30. In a group discussion of mathematics class, four friends Ajay, Binay, Karan and Dinesh are discussing about numbers and quantification. They all are discussing the congruence modulo and different arithmetic functions such as divisor function, sigma function, omega function, Euler's totient function and mobius function. While discussing, they did not agree each other.

According to their understanding related to the topic numbers and quantification, answer the following questions:

(i) Ajay says, mathematical form of sigma function is $\sum_{d/n} d$
Binay says, mathematical form of sigma function is $\sum_{d/n} 1$
Karan says, mathematical form of sigma function is $\sum_{n/d} d$
Dinesh says, mathematical form of sigma function is $\sum_{n/d} 1$
Who is correct ?
(a) Ajay (b) Binay (c) Karan (d) Dinesh

(ii) Ajay says, mathematical form of divisor function is $\sum_{d/n} d$
Binay says, mathematical form of divisor function is $\sum_{d/n} 1$
Karan says, mathematical form of divisor function is $\sum_{n/d} d$
Dinesh says, mathematical form of divisor function is $\sum_{n/d} 1$
Who is correct?
(a) Ajay (b) Binay (c) Karan (d) Dinesh

(iii) Euler's totient function $\emptyset(n) = n - 1$, if and only if n is:
(a) Odd (b) even (c) prime (d) odd prime

(iv) Let m be a fixed positive integer. Then an integer a is congruent to an integer b under modulo m, written as $a \equiv b(mod m)$, if
(a) $a|(m+b)$ (b) $m|(a-b)$ (c) $m|(b-a)$ (d) both b and c.

(v) Ajay says, congruence modulo is reflexive and symmetric relation.
Binay says, congruence modulo is neither reflexive nor symmetric relation.
Karan says, congruence modulo is an equivalence relation.
Dinesh says, congruence modulo is reflexive and symmetric but not a transitive relation.
Who is correct?
(a) Ajay (b) Binay (c) Karan (d) Dinesh

Answers: 1. (c) 2. (b) 3. (a) 4. (d) 5. (b)
6. (a) 7. (c) 8. (a) 9. (b) 10. (a)

11. (c) 12. (d) 13. (b) 14. (a) 15. (c)
16. (b) 17. (b) 18. (c) 19. (b) 20. (d)
21 (a) 22(d) 23(b) 24(b) 25(b)
26(a) 27(a) 28(a) 29 (i)(b) (ii)(a)
(iii)(a) (iv)(c) (v)(a) 30 (i)(a) (ii)(b)
(iii)(c) (iv)(d) (v)(c)

MCQ's FROM MATRICES

APPLIED MATEMATICS

1. If A is a square matrix such that (A – 2I) (A + I) = 0, then A^{-1} is

(a) $\frac{A-I}{2}$ (b) $\frac{A+I}{2}$

(c) 2 (A – I) (d) 2A + I

2. If A = $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ then A^2 + 2A equals

(a) 4A (b) 3A

(c) 2A (d) A

3. If $\begin{bmatrix}x+y & 2x+z\\x-y & 2z+w\end{bmatrix} = \begin{bmatrix}4 & 7\\0 & 10\end{bmatrix}$, then the values of x, y, z and w respectively are

(a) 2, 2, 3, 4 (b) 2, 3, 1, 2

(c) 3, 3, 0, 1 (d) None of these

4. If A = $[a_{ij}]$ is a matrix of order 2, where $a_{ij}=\frac{(i+2j)^2}{2}$, then A is equal to

(a) $\begin{bmatrix}9 & 25\\8 & 18\end{bmatrix}$ (b) $\begin{bmatrix}\frac{9}{2} & \frac{25}{2}\\8 & 18\end{bmatrix}$

(c) $\begin{bmatrix}9 & 25\\4 & 9\end{bmatrix}$ (d) $\begin{bmatrix}\frac{9}{2} & \frac{15}{2}\\4 & 9\end{bmatrix}$

5. A square matrix A = $[a_{ij}]$ of order n is called a lower triangular matrix if a_{ij} = 0 for

(a) i = j (b) i < j

(c) i > j (d) None of these

6. A square matrix A = $[a_{ij}]$ of order n is called a diagonal matrix if a_{ij} = 0 for

(a) i = j (b) i < j

(c) i > j (d) i≠ j

7. For any square matrix A, AA^T is a

(a) Unit matrix (b) symmetric matrix

(c) skew-symmetric matrix (d) diagonal matrix

8. If a matrix A is both symmetric and skew-symmetric, then

(a) A is a diagonal matrix (b) A is zero matrix

(c) A is a scalar matrix (d) A is square matrix

9.If A = diag(3, -1), then matrix A is

(a) $\begin{bmatrix}0 & 3\\0 & -1\end{bmatrix}$ (b) $\begin{bmatrix}-1 & 0\\3 & 0\end{bmatrix}$

(c) $\begin{bmatrix}3 & 0\\0 & -1\end{bmatrix}$ (d) $\begin{bmatrix}3 & -1\\0 & 0\end{bmatrix}$

10. Total number of possible matrices of order 2 × 3 with each entry 1 or 0 is

(a) 6 (b) 36

(c) 32 (d) 64

11. If A is a square matrix such that A^2=A, then $(I + A)^2 - 3A$ is

(a) I (b) 2A

(c) 3I (d) A

12.If matrices A and B are inverse of each other then

(a) AB = BA (b) AB = BA = I

(c) AB = BA = 0 (d) AB = 0, BA = I

13. If A = $\begin{bmatrix}0 & 2\\2 & 0\end{bmatrix}$, $then\ A^2 is$

(a) $\begin{bmatrix}0 & 4\\4 & 0\end{bmatrix}$ (b) $\begin{bmatrix}4 & 0\\4 & 0\end{bmatrix}$

(c) $\begin{bmatrix}0 & 4\\0 & 4\end{bmatrix}$ (d) $\begin{bmatrix}4 & 0\\0 & 4\end{bmatrix}$

14. If A = $\begin{bmatrix}5 & x\\y & 0\end{bmatrix}$ and A = A' then

(a) x = 0, y = 5 (b) x = y

(c) x + y = 5 (d) x – y = 5

15. If A = $\begin{bmatrix}3 & -1 & 2\\4 & 5 & 9\\1 & 3 & 4\end{bmatrix}$, then the value $3a_{22} - 4a_{33}$ is

(a) 1 (b) -1

(c) 0 (d) 2

16. If A = $\begin{bmatrix}0 & a\\0 & 0\end{bmatrix}$ then A^{16} is

(a) $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ (b) $\begin{bmatrix}0 & a\\0 & 0\end{bmatrix}$

(c) $\begin{bmatrix}0 & a\\a & 0\end{bmatrix}$ (d) None of these

17. If a matrix A= $\begin{bmatrix}0 & 2\\0 & 0\end{bmatrix}$ and f(x) = 1+x+$x^2 + x^4 + x^8 + x^{16}$ then f(A) is

(a) $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ (b) $\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$

(c) $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ (d) None of these

18. If A = $\begin{bmatrix}0 & 2\\3 & -4\end{bmatrix}$ $and\ kA = \begin{bmatrix}0 & 3a\\2b & 24\end{bmatrix}$, then the values of k, a and b are respectively

(a) -6, -4, -9 (b) -6, 12, 18

(c) -6, -12, -18 (d) -6, 4, 9

19. If the matrix $A = \begin{bmatrix}0 & a & -3\\2 & 0 & -1\\b & 1 & 0\end{bmatrix}$ is skew symmetric matrix then

(a) a = 0, b = 5 (b) a = -2, b = 3

(c) a = 2, b = -3 (d) None of these

20. If A is matrix of order m × n and B is a matrix such that A$B$and BA^T are both defined, then order of matrix B is

(a) m × m (b) n × n

(c) n × m (d) m × n

21. The matrix A = $\begin{vmatrix}3 & -2 & 7\\-2 & 1 & -3\\7 & -3 & 5\end{vmatrix}$ is a

(a) Symmetric Matrix (b) Skew-Symmetric matrix

(c)Diagonal Matrix (d) Scalar Matrix

22. If $A = \begin{bmatrix}cos\alpha & -sin\alpha\\sin\alpha & cos\alpha\end{bmatrix}$, then A+$A^T$= I then the value of α is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
(c) π (d) None of these

23. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ Is such that $A^2 = I$ then
(a)$1+\alpha^2 + \beta\gamma = 0$ (b)$1-\alpha^2 + \beta\gamma = 0$
(c)$1-\alpha^2 - \beta\gamma = 0$ (d)$1+\alpha^2 - \beta\gamma = 0$

24. The matrix A = $\begin{vmatrix} 0 & -5 & 7 \\ 5 & 0 & 3 \\ -7 & -3 & 0 \end{vmatrix}$ is a
(a) Symmetric Matrix (b) Skew-Symmetric matrix
(c)Diagonal Matrix (d) Scalar Matrix

25. The matrix A = $\begin{bmatrix} 0 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 0 \end{bmatrix}$ is a
(a) Scalar matrix (b) Diagonal matrix
(c) Unit matrix (d) Square matrix

26. If A and B are 2 × 2 matrices, then which of the following is true?
(a) $(A + B)^2 = A^2 + B^2 + 2AB$ (b) $(A - B)^2 = A^2 + B^2 - 2AB$
(c) (A – B) (A + B) = A^2 + AB – BA – B^2 (d) (A + B) (A – B) = A^2 – B^2

27. If A = $\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$ and $A+A^T = I_2$ then α is
$(a)\alpha = n\pi\ n \in Z$ $(b)(2n+1)\frac{\pi}{2}\ n \in Z$
$(c)\alpha = 2n\pi + \frac{\pi}{3}\ n \in Z$ (d)None of these

28. If A = $\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ then A^{4n} where n is a natural number is equal to
(a) I (b) – A
(c) – I (d) A

29. If $\begin{bmatrix} x+y+z \\ x+y \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ then the value of (x, y, z) is
(a) (4, 3, 2) (b) (3, 2, 4)
(c) (2, 3, 4) (d) None of these

30. $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix}$ = 0 then the value of x is
(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$
(c) 1 (d) -1

ANSWERS

S NO	ANS	HINT
1	a	$A^2 - A - 2I = O \Rightarrow A^{-1}(A^2 - A - 2I) = O \Rightarrow A - I - 2A^{-1} = 0$
2	b	$A^2 + A = A + 2A = 3A$, As A is identity matrix so $A^2 = A$
3	a	Solve x + y = 4, 2x+z=7, x – y = 0, 2z+w=10
4	b	$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ find $a_{11}, a_{12}, a_{21}, a_{22}$ using relation $a_{ij} = \frac{(i+2j)^2}{2}$

5	b	All elements above the diagonal elements are zero so, i < j
6	d	All the non diagonal elements are zero i.e. i≠ j
7	b	$(AA^T)^T = (A^T)^T A^T = AA^T$
8	b	$A^T = A \; AND \; A^T = -A \Rightarrow A = -A \Rightarrow 2A = 0$
9	c	
10	d	2^6
11	a	$I + A^2 + 2A - 3A = I + A + 2A - 3A = I$
12	b	
13	d	$A^2 = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$
14	B	$\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix} = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix} \Rightarrow x = y$
15	b	$3a_{22} - 4a_{33}$ =3(5)-4(4)=-1
16	a	$A^2 = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
17	B	As $A^2 = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, so f(A)= I + A
18	a	kA =k $\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$=$\begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix}$=$\begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, and equate corresponding elements
19	b	A' = -A (Skew symmetric matrix)
20	b	
21	a	
22	b	A+$A^T = \begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}\begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix} = \begin{bmatrix} 2cos\alpha & 0 \\ 0 & 2cos\alpha \end{bmatrix}$=$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
23	c	$A^2 = I \Rightarrow \begin{bmatrix} \alpha^2 + \gamma\beta & 0 \\ 0 & \alpha^2 + \gamma\beta \end{bmatrix}$ =$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
24	c	a_{ij}=$-a_{ji}$
25	d	
26	c	AB ≠ BA
27	c	$\begin{bmatrix} 2cos\alpha & 0 \\ 0 & 2cos\alpha \end{bmatrix}$=$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow 2cos\alpha = 1 \Rightarrow \alpha = 2n\pi + \frac{\pi}{3} \; n \in Z$
28	a	I

29	c	Solve: x + y + z = 9, x + y = 5 and y + z = 7
30	b	$[1 \quad x \quad 1]\begin{bmatrix} x-1 \\ 3 \\ -1 \end{bmatrix} = O \Rightarrow [4x-2] = [0] \Rightarrow 4x-2=0 \Rightarrow x = \frac{1}{2}$

QUESTION BANK FOR APPLIED MATHS

CLASS XII

TOPIC: DETERMINANTS

1	$If \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}, then$ (a) $a = \cos 2\theta\, , b = \sin 2\theta$ (b) a =1, b = 1 (c) $b = \cos 2\theta\, , a = \sin 2\theta$ (d) None of these
2	$If\ A = \begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}\ and\ B = \begin{bmatrix} -2 & -2 \\ 0 & -1 \end{bmatrix}\ then (A+B)^{-1}$ (a) *is a skew symmetric matrix* (b) Does not exist (c) $A^{-1} + B^{-1}$ (d) None of these
3	If λ is the determinant of a square matrix A of order n, then the determinant of its adjoint is (a) λ $(b)\lambda^n$ $(c)\ \lambda^{n-1}$ $(d)\lambda^{n+1}$
4	If A and B are any two square matrices of the same order, then (a) (AB)′ = A′B′ (b) adj(AB) = adj(A) adj(B) (c) (AB)′ = B′A′ (d) AB = O ⇒ A = O or B = O
5	If A = $\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, then the value of $\lvert A'A^{-1}\rvert$ is ‘(a) $\cos 4x$ (b) $-\cos 4x$ (c) 0 (d)1
6	A is a non-singular matrix of order 3 such that $A^2 = 3A. then\ the\ value\ of\ \lvert A\rvert\ is$ (a) -3 (b) 3 (c) 9 (d) 27
7	If A is a square matrix of order 3 and $\lvert A\rvert = 5$, the value of $\lvert 2A'\rvert$ is (a) 10 (b) -10 (c) 40 (d)-40
8	If A is a square matrix of order 3 and $\lvert A\rvert = 2, then\ the\ value\ of\ \lvert -AA'\rvert$ is (a) 4 (b)2 (c)-2 (d) -4

9	If matrix A=$\begin{bmatrix} 2 & -1 & 3 \\ \lambda & 0 & 7 \\ -1 & 1 & 4 \end{bmatrix}$ is not invertible for (a) $\lambda = -1$ (b) $\lambda = 1$ (c) $\lambda = 0$ (d) $\lambda \epsilon R - \{1\}$
10	If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0, then\ value\ of\ x\ is$ (a) 3 (b) 0 (c) -1 (d) 1
11	For what value of k inverse does not exist for the matrix $\begin{bmatrix} 3 & 7k \\ 1 & -7 \end{bmatrix}$ (a) 3 (b) -7 (c) -21 (d)-3
12	If A=$\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, then the determinant of the matrix $A^2 - 2A$ (a) 25 (b) -25 (c) 0 (d)4
13	If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & 5 \\ 8 & 3 \end{vmatrix}$, then the value of x is (a)± 3 (b) -3 (c) 3 (d) ± 2
14	If the points (2, -3), (k, -1) and (0, 4) are collinear, then the value of k is (a) 10/7 (b) 7 (c) 10 (d) 0
15	Let $\Delta = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}, where\ 0 \le \theta \le 2\pi, then$ $(a)\Delta = 0$ $(b)\Delta \in (2, \infty)$ $(c)\Delta \in (2, 4)$ $(d)\Delta \in [2,4]$
16	The maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix} (\theta\ is\ real\ number)$ $(a)\frac{1}{2}$ $(b)\ \frac{\sqrt{3}}{2}$ $(c)\sqrt{2}$ $(d)\frac{2\sqrt{3}}{2}$
17	The value of the determinant $\begin{vmatrix} 0 & -a & b \\ a & 0 & c \\ -b & -c & 0 \end{vmatrix}$ is (a) a (b) –a (b)b (d)0
18	If A and B are invertible matrices of same order, then which of the following statements is not true? (a) $\lvert A^{-1}\rvert = \lvert A\rvert^{-1}$ (b) $adj.A = \lvert A\rvert A^{-1}$ (c) $(A+B)^{-1} = B^{-1} + A^{-1}$ (d) $(AB)^{-1} = B^{-1}A^{-1}$
19	If A is a non-singular matrix, then ‘(a) $\lvert A\rvert \neq \lvert A'\rvert$ (b) $\lvert A^{-1}\rvert \neq \lvert A\rvert^{-1}$ (c) $\lvert A\rvert + \lvert A'\rvert \neq 0$ (d) $\lvert AA'\rvert \neq \lvert A^2\rvert$

20	Let f(z)= $\begin{vmatrix} 5 & 3 & 8 \\ 2 & z & 1 \\ 1 & 2 & z \end{vmatrix}$, then f(5) is equal to (a) 10 (b) -20 (c) 80 (d) none of these
21	The value of the determinant $\begin{vmatrix} log_3 512 & log_4 3 \\ log_3 8 & log_4 9 \end{vmatrix}$ *is* (a) 15 (b) 15/2 (c)10 (c) 0
22	If x=-9 is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$, *then the other two roots are* (a) 2, 7 (b)-2, 7 (c) 1, 7 (d) none of these
23	Let A be a non-singular square matrix of order 3 × 3. Then |adj A| is equal to (a) |A| (b)3|A| (c) $\|A\|^3$ (d) $\|A\|^2$
24	If A is an invertible matrix of order 2, then $\|A^{-1}\|$ is equal to (a) $\|A\|$ (b)$\frac{1}{\|A\|}$ (c) 0 (d) 1
25	The sum of the products of elements of any row with the co-factors of corresponding elements is equal to the value of (a) Determinant of matrix (b) 0 (c) 1 (d) None of these
26	Roots of the equation $\begin{vmatrix} 1-x & 2 & 3 \\ 0 & 2-x & 0 \\ 0 & 2 & 3-x \end{vmatrix} = 0$ are (a) 1, 3 (b)1,2, 3 (c) 1, 3 (d) -1, -3
27	If for a matrix A, $A^3 = I$, *then inverse of A is* (a) A (b) A^2 (c) A^3 (d) none of these
28	Which of the following is correct? (a) Determinant is a square matrix (b) Determinant is a number associated to a square matrix. (c) Determinant is a number associated to a matrix. (d) None of these
29.	If $A^2 - A + I = O$, *then inverse of A is equal to* (a) A+I (b) A-I (c) A+2I (d) I-A
30	The adjoint of matrix A = $\begin{bmatrix} 3 & -5 \\ 2 & 1 \end{bmatrix}$ is (a) $\begin{bmatrix} 1 & -5 \\ -2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 2 \\ -5 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix}$

ANSWERS

1	(a) Hint: $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \frac{1}{\sec^2\theta} \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ $\Rightarrow \begin{bmatrix} \frac{1-\tan^2\theta}{\sec^2\theta} & -\frac{2\tan\theta}{\sec^2\theta} \\ \frac{2\tan\theta}{\sec^2\theta} & \frac{1-\tan^2\theta}{\sec^2\theta} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ $\Rightarrow \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$
2	(d) Hint: $A + B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ $\Rightarrow (A+B)^{-1} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$
3	(c) Hint: $\lvert adjA\rvert = \lvert A\rvert^{n-1}$, A is matrix of order n
4	(c)
5	(d) Hint:$A'A^{-1} = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix} \frac{1}{\sec^2 x} \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$ $\Rightarrow A'A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$
6	(d) Hint: $\lvert A^2\rvert = \lvert 3A\rvert \Rightarrow \lvert A\rvert^2 = 27\lvert A\rvert$
7	(c)Hint: $\lvert 2A'\rvert = 2^3\lvert A'\rvert = 8(5) = 40$
8	(d) Hint: $\lvert -AA'\rvert = \lvert -A\rvert\lvert A'\rvert = -2(2) = -4$
9	(b) Hint: $\lvert A\rvert = 0 \Rightarrow -\lambda(-4-3) - 7(2-1) = 0 \Rightarrow \lambda = 1$
10	(c)
11	(d)-3
12	(a) 25
13	(a) ± 3
14	(a) 10/7 Hint: $\begin{vmatrix} 2 & -3 & 1 \\ k & -1 & 1 \\ 0 & 4 & 1 \end{vmatrix} = 0 \Rightarrow 2(-1-4) - k(-3-4) = 0$
15	(d)Hint: $\Delta = 2(1+\sin^2\theta)$ $As\ 0 \le \theta \le 2\pi \Rightarrow 0 \le \sin^2\theta \le 1, \Rightarrow 2 \le (1+\sin^2\theta) \le 4$
16	(a) Hint: $\Delta = -\sin\theta\cos\theta = -\frac{\sin 2\theta}{2}$
17	(d) Hint: determinant of a skew-symmetric matrix of odd order is zero.
18	(c)

19	(c)
20	(c) Hint: f(5)= $\begin{vmatrix} 5 & 3 & 8 \\ 2 & 5 & 1 \\ 1 & 2 & 5 \end{vmatrix} = 5(25-2) - 3(10-1) + 8(4-5) = 80$
21	(b)Hint: $\begin{vmatrix} 9log_3 2 & log_4 3 \\ 3log_3 2 & 2log_4 3 \end{vmatrix} = \begin{vmatrix} \frac{9}{2} log_3 4 & log_4 3 \\ \frac{3}{2} log_3 4 & 2log_4 3 \end{vmatrix} = 9log_3 4 log_4 3 - \frac{3}{2} log_3 4 log_4 3$
22	(a) Hint: $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = x^3 - 67x + 126 = (x+9)(x^2 - 9x + 14)$
23	(d)
24	(b)
25	(a)
26	(b) Hint: $\Delta = (1-x)(2-x)(3-x)$
27	(b)
28	(b)
29	(d) Hint:$A^{-1}(A^2 - A + I) = A^{-1}.O \Rightarrow A - I + A^{-1} = 0$
30	(b)

CALCULUS (APPLIED MATHEMATICS)
MCQ'S

(1.) The maximum value of $\frac{\log x}{x}$ is at

a) x= e b) $x = \frac{1}{e}$ c) x= 1 d) $x = \frac{2}{e}$

(2.) The interval in which the function f(x)=$2x^3$ + $9x^2$ +12x -1 is decreasing

a) [-1,∞) *b*) [-2,-1] c) $(-\infty, -2]$ d) [-1,-1]

(3.) A point on the straight line 2x + 3y = 6 which is closest to the origin

a) $(\frac{2}{13}, \frac{5}{13})$ b) $(\frac{-12}{13}, \frac{18}{13})$ c) $(\frac{12}{13}, \frac{18}{13})$ d) $(\frac{12}{13}, \frac{-18}{13})$

(4.) If f(x)= x^3 − 6 x^2 +18x +5 is an increasing function ∀ x ϵ R . The value of f(x) where the rate of increase is least

a) 25 b) 21 c)15 d)none of these

(5.) The maximum value of $\frac{1}{4x^2+2x+1}$ is at

a) $x = \frac{4}{3}$ b) $x = \frac{2}{3}$ c)$x = 1$ d)$x = \frac{3}{4}$

(6.) If f (x) = $2x^2$- kx + 5 is increasing in [1,2] then k lies in the Interval

(a) (-∞, 4) (b) (4 ,∞) (*c*) (-∞, 8) (d) (8, ∞)

(7.) The function f(x) = ax-b is strictly increasing on R if

(a) a > 0 (b) a < 0 (c) a = 0 (d) none of these

(8.) The function f(x) =tanx –x

a) Always increases b) Always decreases

c) Never increases d) Never decreases

(9.) The smallest value of the polynomial

$x^3 - 18x^2 + 96x$ in [0,9] is

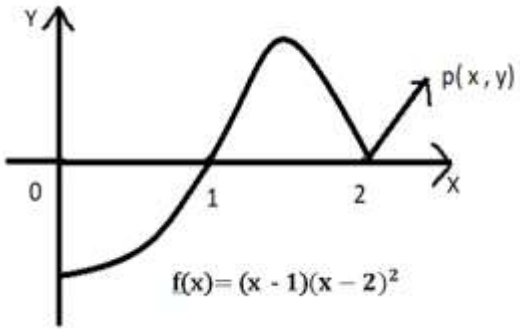
(a) 126 (b) 135 (c)160 (d)0

(10.) Stationary points of f(x) = $3x^4 - 8x^3 + 6x^2$ are

(a) 1, 0 (b) 0, 1 (c) 0,-1 (d) 1, 1

CASE STUDY

A particle is moving along the curve represented by the polynomial f(x) = (x - 1)$(x-2)^2$ as shown in the following figure given below



Based on the above information answer the following questions

11 *The critical points of polynomial f(x) are*

a) $2, -\frac{4}{3}$ *b)* $2, \frac{4}{3}$ *c)* $-2, -\frac{4}{3}$ *d)* $-2, \frac{4}{3}$

12 *The interval where f(x) is increasing is*

a) $(-\infty, \frac{4}{3}] \cup [2, \infty)$ *b)* $(-\infty, -\frac{4}{3}] \cup [2, \infty)$ *c)* $[2,\infty)$ *d)* $(-\infty, \frac{4}{3}]$

13 *The interval where f(x) is decreasing is*

a) $[\frac{4}{3}, 2]$ *b)* $[-\frac{4}{3}, 2]$ *c)* $[0, 2]$ *d)none of these*

14. *What is the minimum value of* ***f(x)= (x - 1)***$(x-2)^2$

(a) 1 *(b) 2* *(c)0* *(d)3*

15. *What is the point of local maxima for*

f(x)= (x - 1) $(x-2)^2$ *?*

(a)2 *(b)* $\frac{4}{3}$ *(c) 0* *(d) – 2*

16. *The equation of the tangent to the curve y = sin x at (0, 0) is*

(a) x + y = 0 *(b) x - y = 0* *(c) 2x + y = 0* *(d) 2x - y = 0*

17. The points at which the tangent to the curve $y = x^3 + 5$ is perpendicular to the line x + 3y = 2 are

(a) (1, 6), (-1, 4) (b) (1, 6), (1, 4) (c) (6, 1), (4, 1) (d) (6, 1), (-1, 4)

18. The point at which the tangent to the curve $y = \sqrt{4x-3}-1$ has its slope 2/3.

(a) (3,3) (b) (3, 2) (c) (2, 3) (d) (2, 2)

19. The lines 3x - 4y + 4 = 0 and 6x - 8y - 7 = 0 are tangents to the same circle. The radius of this circle is:

(a) 3/2 (b) ¾ (c) 4/5 (d) 7/10

20. Differentiate $x^2+y^2=25$ implicitly with respect to x.

(a) **-x/y** (b) **–y/x** (c) **x/2y** (d) **2y/x**

21. **If $x=4z^2+5$ and $y= 6z^2+72z+3$ then $\frac{d^2y}{dx^2}$ is.**

(a) $\frac{-7}{64z^3}$ (b) $\frac{7}{64z^3}$ (c) $\frac{-7}{66z^3}$ (d) *no*ne of these

22. The second order derivative of $\log_e(\log x)$

a). $\frac{-(1+\log x)}{(x\log x)^2}$ **b)** $\frac{(1+\log x)}{(x\log x)^2}$ **c)** $\frac{-(1+\log x)}{(\log x)^2}$ **d)** $\frac{-(1+\log x)}{x}$

23. If $y = a x^{n-1} + b x^{-n}$ then $x^2 \frac{d^2y}{dx^2}$ is

a) n(n+1) y b) n(n-1)y c) ny d) n^2y

24. If $y = ax^2+bx+c$ then $y^3 \frac{d^2y}{dx^2}$ is

a) a constant b) a function of x only c) a function of y only d)a function of x and y.

25. A toy manufacturing firm assesses its variable cost to be 'x' times the sum of 30 and 'x', where 'x' is the number of toys produced, also the cost incurred on storage is Rs 1500. the marginal cost when 20 toys are produced.

a) 60 b) 70 c) 80 d) 90

26. The price per unit of a commodity produced by a company is given by p= 30 -2x and 'x' is the quantity demanded. The marginal revenue when 5 commodities are in demand (or produced).

a)10 b) 20 c) 30 d) 40

27. The cost of manufacturing x units of a certain commodity is $27+15x+3x^2$. The output for which AC is increasing is

a) $x > 3$ b) $x < 3$ c) $x=3$ d) none of these

28. The cost function for a certain commodity is $C(x) = 3+2x -\frac{1}{4}x^2$ .The AVC when 4 items are produced is

a) 1 b)2 c) 4 d) 5

29. The average cost function with producing and marketing x units of an item is AC= 2x-11+50/x. The marginal cost function is

a) 4x+11 b) 4x-11 c)-11 d) none of these

30. The total cost function for x units of a commodity is $C(x) = \frac{x^3}{3} + 3x^2 -7x +16$. The AC is

a) $\frac{x^3}{3} + 3x$ b) $\frac{x^2}{3} +3x-7 +\frac{16}{x}$ c) $7+\frac{16}{x}$ d) $\frac{x^3}{3} + 3x^2 -7x$

ANSWERS

1. let $y = \frac{\log x}{x}$

$\frac{dy}{dx} = \frac{\frac{x}{x} - \log x}{x^2} = \frac{1-\log x}{x^2}$ $\frac{dy}{dx} = 0$ => $\log x = 1$ => $x=e$

$\frac{d^2y}{dx^2} = \frac{x^2\,(\frac{1}{x}) - (1-\log x)\,(2x)}{(x^2)^2} = \frac{-3x + 2x\log x}{(x^2)^2}$ $\frac{d^2y}{dx^2}|_{x=e} = \frac{-3e+2e}{(e^2)^2} = -\frac{e}{e^4} = -\frac{e}{e^2} \leq 0$

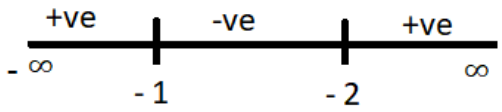
=> y is maximum at x=0 & maximum value $\frac{\log e}{e} = \frac{1}{e}$ **Answer** - Option **b**

2.

$f'(x) = 6x^2 + 18x+12 = 6(x^2 + 3x +2)$

$= 6[(x+1)(x+2)]$

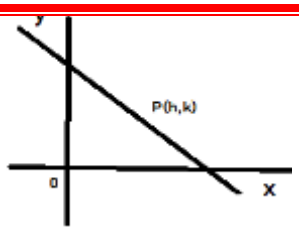
f '(x) =0 =) x= - 1, -2



For increasing $f'(x) \leq 0$

$X \in [-2, -1]$ **Answer** option **b**

3. P (h,k) lies on the line



$2h+ 3k- 6 = 0$ =) $k = \frac{6-2h}{3}$

$|op| = \sqrt{h^2 + k^2}$ =) $|op|^2 = h^2 + k^2 = h^2 + (\frac{6-2h}{3})^2$

$|op|^2$ = f(h) say =) $f(h) = h^2 + (\frac{6-2h}{3})^2$

$f'(h) = \frac{1}{9}[13h^2 - 24h + 36]$ $f'(h) = 0$ =) $h = \frac{12}{13}$

$\frac{f''(h)}{h} = \frac{12}{13} = \frac{26}{9} > 0$ =) f (h) is minimum at $h = \frac{12}{3}$

$K = \frac{6 - 2(\frac{12}{13})}{3} = \frac{18}{13}$ **Answer** Option **c) ($\frac{12}{13}$, $\frac{18}{13}$)**

4. $f(x) = (x^3 - 6x^2 + 18x + 5)$

$f'(x) = 3x^2 - 12x + 18 = 3(x^2 - 4x + 6) = 3[(x-2)^2 + 2] \geq 3(0+2)$ ∀ $x \in R$

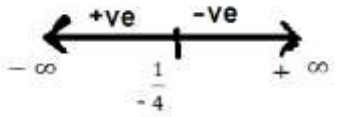
=) $f'(x) \geq 6$ [$\because (x-2)^2 \geq 0$ ∀ $x \in R$] =) $f'(x) > 0$ ∀ $x \in R$

=) f(x) is increasing function ∀ $x \in R$ Now rate of increase is least when

$3[(x-2)^2 + 2]$ is least When $(x-2)^2 = 0$ =) $x = 2$.

f (2) = 25 **Answer** option **a) 25**

5. $f(x) = \frac{1}{4x^2+2x+1}$



$f'(x) = 0$ =) $\frac{-2(4x+1)}{(4x^2+2x+1)^2}$ $f'(x) = 0$ =) $x = -\frac{1}{4}$

=) f(x) is maximum at $x = -\frac{1}{4}$ $f(-\frac{1}{4}) = \frac{4}{3}$ **answer** option **a)$\frac{4}{3}$**

6. $f(x) = 2x^2 - kx + 5$ $f'(x) = 4x - k$

$f'(x) > 0$ $4x - k > 0$ $x > \frac{k}{4}$ $x \in [1,2]$

=) $\frac{k}{4} < 1$ or $\frac{k}{4} < 2$ $K < 4$ $k < 8$ =) $k < 4$

=) $k \in (-\infty, 4)$ **answer** option **(a) $(-\infty, 4)$**

7. $a > 0$ **answer** option **(a) a > 0**

8. $f(x) = \tan x - x$ $f'(x) = sec^2x - 1$

For $x \in R$ =) $f'(x) = tan^2x$

=) $f'(x) \geq 0$ **Answer** option **(A) Always increases**

9. $f(x) = x^3 - 18x^2 + 96x$ $f'(x) = 3x^2 - 36x + 96$

f '(x)= 0 f (4) = 160 f (8)= 128 f(9)= 135

f(x) is minimum at x = 0 in [0 , 9] **Answer** option **(d) 0**

10. f(x)= $3x^4 - 8x^3 + 6x^2$ f '(x) = 0

=) $12(x^3 - 2x^2 + x) = 0$ =) x =0, 1 **Answer** option **(b) (0, 1)**

CASE STUDY ANSWERS

11. f(x)= (x - 1)$(x-2)^2$

f '(x)= (x-1) 2(x-2)+$(x-2)^2$ = (x-2) [2x - 2+ x - 2]

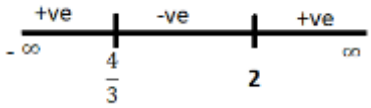
f '(x) = (x-2)(3x-4)

f '(x)=0 => x = 2 , $\frac{4}{3}$ **Answer** option **b) 2, $\frac{4}{3}$**

12. f '(x)= 0

(X-2) (3x-4)=0

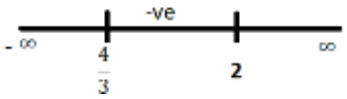
X = $\frac{4}{3}$ **, 2**



f(x) is increasing in **Answer** option **(a) (- ∞ , $\frac{4}{3}$] U [2,∞)**

13. from B) critical points are - x= $\frac{4}{3}$, 2

f(x) in [$\frac{4}{3}$, 2] **answer** option **a)[$\frac{4}{3}$, 2]**



14. Critical points are x=2, $\frac{4}{3}$

f ' (x)=(x-2)(4x-3)= $4x^2$- 11x + 6 f ''(x)= 8x – 11

at x=2 , f ''(2) = 8 (2) – 11 > 0 => f(x) has local minima at x = 2

Local minimum value for f(2)= (2 - 1)$(2-2)^2$ = 1 **Answer** option (a) 1

15. At x = $\frac{4}{3}$

f''($\frac{4}{3}$) = 8 ($\frac{4}{3}$) − 1 = $\frac{32-33}{3}$ = - $\frac{1}{3}$ < 0

=> f(x) has local maxima at x= $\frac{4}{3}$ **Answer** option **(b) $\frac{4}{3}$**

16.Here, y = sin x dy/dx = cos x dy/dx at x=0 is 1

So, slope = 1 ∴ Equation of tangent to the curve is y - 0 = 1(x - 0)

⇒ y = x ⇒ x - y = 0

17. Tangent to the curve $y = x^3 + 5$ $m_1=3x^2$ ---(i)

Line equation 3y = 2 – x ⇒ y=−1/3x+2 m_2=−1/3 ---(ii)

Now, $m_1 \times m_2$ = -1 (for lines to be perpendicular to (ii)] $3x^2 \times (-1/3) = -1$

$x^2 = 1$ **x = ± 1** **Now,** y = 1 + 5 y = 6 y = -1 + 5 = 4

so Points at which these lines are perpendicular are (1, 6), (-1, 4)

18. Let us first find out f'(x) for the curve y = f(x) = √4x−3 −1. f'(x) = $\frac{d}{dx}$(√4x−3 −1) =2/3

⇨ X=3 then y=2. ∴ The required point is = **(3, 2)**.

19. Since a1/a2=b1/b2≠c1c2, the given pair of lines are parallel to each other.

Multiplying the first equation by 2, the two equations can be re-written as 6x - 8y + 8 = 0 and 6x - 7y - 7 = 0. The distance between them will be:

Distance=$|c1-c2|/\sqrt{a^2+b^2}=|8-(-7)|\sqrt{6^2+(-8)^2}=15/\sqrt{100}=15/10=3/2$.

This distance is the diameter of the circle: Therefore, the Radius=$1/2\times(1/2)=3/4$.

20. (a) 21(a) 22. (a) 23. (a) 24. (c)

25. The total cost function C(x) is given by,

$C(x) = x(x + 30) + 1500 = x^2 + 30x + 1500$ MC is given by, MC= dc/dx= 2x+30

Marginal cost of producing 20 toys is $MC(20) = dC/dx]_{x=20} = 2(20)+ 30 = 70$ Ans

26. Revenue function is, $R= px= (30x - 2x^2)$ MR= 30-4x

MR when 5 commodities are in demand is Rs 10.

27. $x > 3$ 28. 1 29. 4x -11 30. $\frac{x^2}{3}+3x-7+\frac{16}{x}$

PROBABILITY (MCQ's)

Q.1 Let x be a discrete random variable whose probability distribution is defined as follows:

$$P(X=x) = \begin{cases} k(x+1), & for\ x = 1,2,3,4 \\ 2kx, & for\ x = 5,6,7 \\ 0, & otherwise \end{cases}$$

Where 'k' is constant. Find k.

(a) 1/50 (b) 2/50 (c) 3/50 (d) 4/50

Q.2 For the following probability distribution:

X	-4	-3	-2	-1	0
P(X)	0.1	0.2	0.3	0.2	0.2

The value of E(X) is.

(a) 0 (b) -1 (c) -2 (d) -1.8

Q.3 For the following probability distribution:

X	1	2	3	4
P(X)	1/10	1/5	3/10	2/5

The value of $E(X^2)$ is.

(a) 5 (b) 10 (c) 15 (d) 20

Q.4 A random variable X takes the values 0, 1, 2, 3 and its mean is 1.3. If P(X=3) = 2P(X=1) and P(X=2) = 0.3, then P(X=0) is

(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

Q.5 For 6 trials of an experiment, let X be a binomial variable which satisfies the relation 9P(X=4) = P(X=2). Find the probability of success.

(a) 1/4 (b) 1/3 (c) 1/2 (d) 1/9

Q.6 Which of the following is true:

(a) Mean < Variance (b) Mean > Variance

(c) Mean = Variance (d) None of these

Q.7 If the probability of defective bolts is 0.1, find the mean and standard deviation for the distribution of defective bolts in a total of 500 bolts.

(a) 4.71 (b) 5.71 (c) 6.71 (d) 7.71

Q.8 What is the expectation of the number of heads in 15 tosses of a coin?

(a) 4.7 (b) 5.7 (c) 6.7 (d) 7.5

Q.9 if two dice are rolled12 times, obtain the mean and the variance of the distribution of success, if getting a total greater than 4 is considered a success.

(a) 10, 5/3 (b) 15, 3/5 (c) 10, 3/5 (d) 15, 5/3

Q.10 Which one is not a requirement of a binomial distribution?

(a) There are 2 outcomes for each trial

(b) There are a fixed number of trials

(c) The outcomes must be dependent on each other

(d) The probability of success must be the same for all the trials.

Q.11 In a binomial distribution, the probability of getting success is ¼ and standard deviation is 3. Then, its mean is

(a) 6 (b) 8 (c) 12 (d) 10

Q.12 Let X denotes the number of times heads occur in n tosses of a fair coin. If P(X=4), P(X=5) and P(X=6) are in AP; the value of n is

(a) 7, 14 (b) 10, 14 (c) 12, 7 (d)14, 12

Q.13 if in a binomial distribution n = 4, P(X=0) = 16/81, then P(X=4) equals

(a) 1/16 (b) 1/81 (c) 1/27 (d)1/8

Q.14 If X is a binomial variable with parameters n and p, where 0<p<1 such that $\frac{P(X=r)}{P(X=n-r)}$ is independent of n and r, then p is equal to

(a) 1/2 (b) 1/3 (c) 1/4 (d)1/8

Q.15 the probability of selection a male or a female is same. If the probability that in an office of n persons (n-1) males being selected is $\frac{3}{2^{10}}$, the value of n is

(a) 5 (b) 3 (c) 10 (d)12

Q,16 If the probability that an individual suffers a bad reaction from injection of a given serum is 0.001, determine the probability that out of 2000 individuals exactly 3 individuals will suffer from a bad reaction(use e^{-2}=0.1353):

(A) 0.18 (B) 1.18 (C) 0.195 (D) 2.180

Q.17 Suppose that X has a Poisson distribution. If P(X=2)=$\frac{2}{3}$ P(X=1) , then the value of P(X=0) is:

(A) e^{-4} (B) $e^{-4/3}$ (C) $e^{-2/3}$ (D) $e^{4/3}$

Q.18 If X is a normal distribution random variable with mean μ=10 and standard deviation σ=2, then value of P(X< 13) is:

(A) 0.0014 (B) 0.993 (C) 1.5 (D) 0.9332

Q.19 The monthly salaries of workers in a certain factory are normally distributed. The mean salary is Rs. 4000 and standard deviation is Rs. 450. If 668 workers are getting salary less than Rs. 3325, the total number of workers in the factory are:

(A) 1000 (B)10000 (C)45000 (D)15000

Q.20 If the variance of a Poisson distribution is 2 , then P(X=2) is:

(A) $\frac{2}{e^2}$ (B) $2e^2$ (C) $\frac{4}{e^2}$ (D) $4e^2$

Q.21 If Z is a standard normal variable , then P(0< Z < 1.7) is equal to:

(A) F(0) – F(1.7) (B) F(1.7) – F(0) (C) 1 - F(1.7) (D) F(1.7) – 1

Q.22 If X is a Poisson variable such that P(X=k) = P(X=k+1) , then variance of X is:

(A) k – 1 (B) k (C) k + 1 (D) k + 2

Q.23 The area under the standard normal curve which lies to the right of z = - 0.66 is:

(A) 1 - F(0.66) (B) F(0.66) - 1 (C) F(0.66) (D) F(-0.66)

Q.24 If F(Z) is a cumulative distribution function , then the value of F(∞) is:

(A) 0 (B) 1 (C) 2 (D) 0.5

Q.25 If X is normally distributed with mean 20 and standard deviation 4, then standard normal variable Z corresponding to X = 21 is:

(A) 1.25 (B) – 1.25 (C) – 0.25 (D) 0.25

Q.26-30**(Case study question)**: An urn contains 25 balls of which 10 balls bear a mark X and remaining 15 bear a mark Y. A ball is drawn at random from the urn, its mark noted down and it is replaced. In this way 6 balls are drawn.

Based on the above information, answer the following questions (26-30):

Q.26 The probability that all balls will bear X mark is

(A) $\left(\frac{3}{5}\right)^6$ (B) $\left(\frac{2}{5}\right)^6$ (C) 1 - $\left(\frac{3}{5}\right)^6$ (D) 1 - $\left(\frac{2}{5}\right)^6$

Q.27 The probability that all balls will not bear X mark is

(A) $\left(\frac{3}{5}\right)^6$ (B) $\left(\frac{2}{5}\right)^6$ (C) 1 - $\left(\frac{3}{5}\right)^6$ (D) 1 - $\left(\frac{2}{5}\right)^6$

Q.28 The probability that at most 2 balls will bear mark Y is

(A) $7\left(\frac{2}{5}\right)^4$ (B) $7\left(\frac{3}{5}\right)^4$ (C) $7\left(\frac{2}{5}\right)^2$ (D) $7\left(\frac{3}{5}\right)^2$

Q.29 The probability that at least 2 balls will bear mark Y is

(A) $\frac{14652}{15625}$ (B) $\frac{2997}{3125}$ (C) $\frac{2897}{3125}$ (D) $\frac{14562}{15625}$

Q.30The probability that the number of balls with X mark and Y mark will be equal is

(A) $\frac{684}{3125}$ (B) $\frac{486}{3125}$ (C) $\frac{864}{3125}$ (D) $\frac{486}{625}$

Answer PROBABILITY

TOPIC- DISCRETE TRANDOM VARIABLE AND ITS PROBABILITY DISTRIBUTION

Q.No.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
Ans.	(a)	(d)	(b)	(d)	(a)	(b)	(c)	(d)	(a)	(c)
Q.No.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
Ans.	(c)	(a)	(b)	(a)	(d)	(a)	(b)	(d)	(b)	(a)
Q.No.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
Ans.	(b)	(c)	(c)	(b)	(d)	(b)	(d)	(a)	(b)	(c)

Ans.1

The sum of the probabilities in a probability distribution is 1.

So, P(x=10 + P(x=2) +P(x=3) +P(x=4)+P(x=5)+P(x=6)+P(x=7) = 1

So, 2k + 3k + 4k + 5k + 10k + 12k + 14k = 1 or 50k = 1 0r k = 1/50 Ans.

Ans.2

Since, E(X) = X.P(X) = (-4)x(0.1) + (-3)x(0.2) + (-2)x(0.3) + (-1) x(0.2) + (0) x (0.2)

= -1.8 Ans.

Ans.3

Since, $E(X^2) = \sum P.X^2 = \frac{1}{10}.1^2 + \frac{1}{5}.2^2 + \frac{3}{10}.3^2 + \frac{2}{5}.4^2 = \frac{100}{10} = 10\ Ans.$

Ans.4

Let P(X=0) = x and P(X=1) = p, so P(X=3) = 2p

X :	0	1	2	3
P(X):	x	p	0.3	2p

$$E(X) = \sum X.P(X) = 0(x) + 1(p) + 2(0.3) + 3(2p)$$

1.3 = 7p + 0.6 p = 0.1 Since $\sum P(X) = 1$

So, x + p + 0.3 + 2p = 1 x = 0.7 – 3p = 0.7 – 0.3 = 0.4 Ans.

Ans.5

Since $P(X = r) = {}^{n}_{r}Cp^{r}q^{n-r}$

$P(X = r) = {}^{6}_{r}Cp^{r}q^{6-r}$ Since n = 6.

It is given that

9P(X=4) = P(X=2) $9{}^{6}_{4}Cp^{4}q^{2} = {}^{6}_{2}Cp^{2}q^{4}$ or $9p^2 = q^2$

So, 3p = q 3p = 1-p or p = ¼. Ans.

Ans. 6

Let X be a binomial variable with parameters n and p. Then,

Mean = np and Variance = npq

Mean – Variance = np – npq = np(1 – q) = np² $(n \in N, p > 0 \ \therefore np^2 > 0)$

Mean – Variance > 0 Mean > Variance. Ans.

Ans. 7

We have n = 500 and p = 0.1 Mean =np = 500x0.1 = 50

SD= $\sqrt{Variance} = \sqrt{npq} = \sqrt{500X0.1X0.9} = 6.71$

Ans. 8

Let p be the probability of getting a head in a single toss. Then, p = ½ and n = 15

So, expectation = E(X) = np = 15x1/2 = 7.5 Ans.

Ans.9

Given n = 12 and let p = probability of getting a total greater than 4 in a single throw of a pair of dice.

p = 1 – probability of geeting a totqal less than or equal to 4

p = 1 – 6/36 = 5/6 or q = 1 – p = 1 – 5/6 = 1/6

Mean = np = 12x5/6 = 10 Ans. Variance = npq = 12x5/6x1/6 = 5/3 Ans.

Ans. 10

The outcomes are independent in binomial distribution.

Ans.11

Given p = ¼ so, q = 1 – p = 1- ¼ = ¾ and S.D. = 3

S.D. = $\sqrt{npq} = \sqrt{Mean\ X\ q}$ $3 = \sqrt{Mean\ X\ q}$ Mean = 3^2 / q = 9x4/3 = 12 Ans.

Ans. 12

Since, P(X=4), P(X=5) and P(X=6) are in AP

So 2.P(X=5) = P(X=4) + P(X=6)

$2.\ {}^{n}_{5}Cp^{5}q^{n-5} = {}^{n}_{4}Cp^{4}q^{n-4} + {}^{n}_{6}Cp^{6}q^{n-6}$

$2.\ \frac{n!}{(n-5)!5!}pq^{-1} = \frac{n!}{(n-4)!4!}q^{-2} + \frac{n!}{(n-6)!6!}p^2$

$\frac{2pq^{-1}}{(n-5).5} = \frac{q^{-2}}{(n-4)(n-5).5} + \frac{p^2}{6.5}$ since p = q = ½ From this n = 7, 14 Ans.

Ans. 13

Given n = 4, P(X=0) = 16/81 ${}^{4}_{0}Cp^{4}q^{0} = \frac{16}{81}$

$p^4 = \frac{16}{81}$ so, p = 2/3 and q = 1 – p = 1/3 P(X=4) = ${}^{4}_{4}Cp^{0}q^{4} = q^4 = \left(\frac{1}{3}\right)^4 = \frac{1}{81}$ Ans.

Ans. 14

Given that $\frac{P(X=r)}{P(X=n-r)}$ is independent of n and r and

$\frac{{}^{n}_{r}Cp^{n-r}q^{r}}{{}^{n}_{n-r}Cp^{r}q^{n-r}} = \left(\frac{p}{q}\right)^{n-2r}$ since it is independent of n and r so, n – 2r = 0

So, p/q = 1 or p = q so, p = ½ Ans.

Ans. 15

Given P(male) = P(female) = ½ = p = q. And P(X=n-1) = $\frac{3}{2^{10}}$

${}^{n}_{n-1}Cp^{n-(n-1)}q^{n-1} = \frac{3}{2^{10}}$

$\frac{n!}{(n-(n-1))!(n-1)!}pq^{n-1} = \frac{3}{2^{10}}$ $n\frac{1}{2}\left(\frac{1}{2}\right)^{n-1} = \frac{3}{2^{10}}$

$\frac{n}{2^{n}} = \frac{3}{2^{10}} = \frac{12}{2^{12}}$ so, n = 12 Ans.

Ans. 16-

N=2000, p=0.001 ,r=3 then λ=np=2000x0.001=2

Using Poisson approximation, P(X=r)=$\frac{\lambda^{r}e^{-\lambda}}{r!}$

P(3)= $\frac{2^{3}e^{-2}}{3!}$ =0.180 Correct option is (A)

Ans 17

Given that P(X=2)=$\frac{2}{3}$ P(X=1) $\therefore \frac{\lambda^{2}e^{-\lambda}}{2!} = \frac{2}{3}\frac{\lambda^{1}e^{-\lambda}}{1!}$

λ=$\frac{4}{3}$ hence, P(X=0)=$e^{-4/3}$ Correct option is (B)

Ans 18

Given that μ=10 , σ=2 then Z= $\frac{X-10}{2}$ for X=13 , Z= $\frac{13-10}{2}$ = 1.5

$\therefore$ P(X< 13)=P(X< 1.5)= F(1.5)= 0.9332 Correct option is (D)

Ans 19

Given that μ=4000 , σ=450 then Z= $\frac{X-4000}{450}$

for X=3325, Z= $\frac{3325-4000}{450}$ = - 1.5

$\therefore$ P(X< −1.5)=P(X< −1.5)= F(-1.5)= 1 – F(1.5) = 1 -0.9332=0.0668

Now , 0.0668 x n = 668 then n = 10000 Correct option is (B)

Ans 20

Given λ= 2 , X=r=2

P(X=2)= $\frac{2^{2}e^{-2}}{2!} = \frac{2}{e^{2}}$ Correct option is (A)

Ans 21

P(0< Z < 1.7) = F(1.7) – F(0) Correct option is (B)

Ans 22

P(X=k)= P(X=k+1) then $\frac{\lambda^{k}e^{-\lambda}}{k!} = \frac{\lambda^{k+1}e^{-\lambda}}{(k+1)!}$ Then λ= k+1 Correct option is (C)

Ans 23

Correct option is (C) F(0.66)

Ans 24

Correct option is (B) : F(∞) = 1

Ans 25 for X=21 , Z= $\frac{21-20}{4}$ = 0.25 Correct option is (D)

INDEX NUMBERS AND TIME BASED SERIES (MCQ's)

1. Index numbers are free from a unit of measurement because the index number shows
   (a) Average changes (b) Variations
   (c) Relative Changes (d) None of these.
2. Base period always –

(a) Normal Period
(b) Abnormality should be present in the base period
(c) Base period should be far ago
(d) None of the above

3. Simple average of relatives is equal to :-
   (a) $\frac{P_1}{P_0} \times 100$
   (b) $\frac{\sum P_1}{\sum P_0} \times 100$
   (c) $\sum\left(\frac{P_1}{P_0}\right)$
   (d) $\frac{1}{N}\sum\left(\frac{P_1}{P_0}\right) \times 100$

4. If all the values are not equal importance the index number is called :
   (a) Simple
   (b) Unweighted
   (c) Weighted
   (d) None
5. The most appropriate average in averaging the price relatives are:-
   (a) Median
   (b) Harmonic mean
   (c) Arithmetic mean
   (d) Geometric mean
6. Index Number for base year is always considered as ----
   (a) 100 (c) 201
   (b) 101 (d) 1000
7. Index number is a special type of
   (a) Average
   (b) Dispersion
   (c) Correlation
   (d) None of these
8. The prices of a commodity in the year 1975 and 1980 were 25 and 30 respectively taking 1980 as base year the price relative is :-
   (a) 110.25
   (b) 113.25
   (c) 109.78
   (d) None of these
9. From the following data for the 5 groups combined

Group	Weight	Index No.
Food	35	425
Cloth	15	235
Power & Fuel	20	215
Rent & Rates	8	115

Miscellaneous	22	150

The general index number is:-

(a) 270 (c) 268.5

(b) 269.2 (d) 272.5

10. Index numbers are use for
    (a) Forecasting
    (b) Fixed prices
    (c) Different prices
    (d) Constant prices
11. A fire in a factory delaying production for some time is
    (a) Long term tend
    (b) Cyclic trend
    (c) Seasonal trend
    (d) Irregular trend
12. Price relative of sugar is 110 in 2002 Compared to 2001. If sugar costs Rs. 16.50 per kg in 2002 what did it cost in 2001?
    (a) Rs. 15 per Kg. (c) Rs. 14.85 per Kg.
    (b) Rs. 18.5 per Kg. (d) Rs. 13.75 per Kg.
13. Seasonal Variations are
    (a) Short term
    (b) Long term
    (c) Sudden
    (d) None of these
14. For the given five values 15, 24, 18, 33, 42 the three years moving averages are:
    (a) 19, 22, 33
    (b) 19, 25, 31
    (c) 19, 30, 31
    (d) 19, 25, 33
15. If Laspeyre's index number = 160, Paasche's index number = 90 then Fisher's index number is
    (a) 90
    (b) 120
    (c) 140
    (d) 160

16. The time reversal test is satisfied by

(a)Simple aggregative method

(b) Fisher's method

(c)both (a) & (b)

(d) None of these

17. The time reversal test is not satisfied by

(a)Simple aggregative method (c)Paasche's method

(b) Laspeyre's method (d)both (b) & (c)

18. If the index number of current year is computed on the basis of base year (P_{01}) and again the index number of base year is computed on the basis of current year (P_{10}) then

(a) $P_{01} \times P_{10} = 1$

(b) $P_{01} \times P_{10} = 2$

(c) $P_{01} \times P_{10} = ½$

(d)None of these

19. A small industrial concern used three raw materials A,B and C in its manufacturing process.The prices of the materials was as shown below

Commodities	Price in Rs. in the year 2005	Price in Rs. in the year 2015
A	4	5
B	60	57
C	36	42

Using 2005 as the base year ,then a simple aggregate price index for 2015 is

(a)102

(b)106

(c)107

(d)104

20. A fire in a factory delaying production for some time is

(a)Long term trend (c) Seasonal trend

(b)Cyclical trend (d) Irregular trend

21. The rise in prices before Diwali is an example of

(a)Seasonal trend (c)Seasonal trend

(b)Cyclical trend (d)Irregular trend

22. Time series analysis helps to

(a)Plan future operations

(b)Understand the behaviour of a variable in the past.

(c)Predict the future behaviour of a variable

(d)All of the above.

23. Irregular variations in a time series are caused by
(a) Lockouts and strikes (b) Epidemics
(c) Floods (d) All of these

24. If the price index is 132, It means that price has increased by ……compared to base period
(a) 33% (b) 32%
(c) 13% (d) 8%

25. If the price index is 87, It means that price has decreased by ……compared to base period
(a) 33% (b) 32%
(c) 13% (d) 8%

26. Seasonal variations are
(a) Short term (c) Sudden
(b) Long term (d) None of thes

27. Which index number is called ideal index number?
(a) Laspeyre's (b) Paasche's
(c) Fisher's (d) None of these

28. If $\sum p_1 q_0 = 623$,$\sum p_1 q_1 = 517, \sum p_0 q_0 = 584$, $\sum p_0 q_1 = 484$ then Fisher's price index number(P_{01}) (approx.)is
(a)106.23 (b) 106.52
(c) 106.74 (d)106.34

29. Unit test is not satisfy by
(a)Simple aggregative method (b) Laspeyre's method
(c) Paasche's method (d) None of these

30. The Paasche's index number is
(a)$P_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$ (b)$P_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$
(c)$P_{01} = \frac{\sum p_1 w}{\sum p_0 w} \times 100$ (d) None of these

Answers

1. (c), Index numbers are unit – free measurement of economic indicators. Index numbers are based on a value of 100, which makes it easy to measure percent changes
2. (a) 3. (d) 4.(c) 5.(d) 6. (a) 7.(a)
8. (d), $\frac{P_n}{P_o} \times 100$, Then $\frac{25}{30} \times 100 = 83.33$
9. (b) Group Weight index number $= \frac{\text{Weighted index}}{\text{Total Weight}} = \frac{26920}{100} = 269.2$
10. (a) 11. (d) 12. (a) 13.(a) 14.(b) 15.(b)
16.(c) 17.(d) 18.(a)
19.(d) find$\sum p_0$, $\sum p_1$ & using $P_{01} = \frac{\sum p_1}{\sum p_0} \times 100$
20.(d) 21.(a) 22.(d) 23.(d) 24.(b) 25.(c)
26.(a) 27.(c) 28.(c),Using Fisher's price index formula
29.(a) 30.(b)